Sample IVMO Intermediate
Time allowed - 1 Hour

1. $4.321+43.21+432.1+4321$

2. Draw a ring round the number below that is divisible by 18.

834257

764522

427536

869365

647381

3.
$$\begin{array}{r} 683 \\ \times 996 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 1756 \\ \times 1003 \\ \hline \end{array}$$

5. 7.5^2

6. 47×49

7. Divide, leaving the remainder as a whole number.

$$97 \overline{)\,21311}$$

8. Divide,

$$113 \overline{)\,25998}$$

9. 43^3

10. Find the square root of the perfect square, 5776.

11. 44×404

12. 3.2% of 25

13. 993^2

14. 52.17×2.823

15. Convert $\frac{116}{125}$ to decimal.

16. Convert the fraction, $\frac{7}{19}$, to decimal, correct to 9 decimal places.

17. Find the density of a lump of Protactinium that has a mass of 48.96 grams and a volume of $3.2\,cm^3$.

18. Convert $0.3\dot{2}\dot{5}$ to a fraction in lowest terms.

19. Solve the simultaneous equations,

$$11x + 3y = 15$$
$$-13x + 6y = 30$$

20. Solve,

$$\frac{4x}{5} - \frac{2x}{3} = 4$$

21. If $6x - y = 21$ and $6y - x = 14$, what is the value of $x - y$?

22. Find the minimum value of the function,

$$f(x) = x^2 - 8x + 23$$

23. Solve by factorisation,

$$5x^2 + 17x - 12 = 0$$

24. Given that $(x-2)$ is a factor of,

$$f(x) = 6x^3 - 19x^2 + 9x + 10$$

find the solutions to $f(x) = 0$.

25. Find the equation of the straight line with gradient, 3, and that passes through the point (4, 2).

26. Two lines have equations, $2x + 3y = 15$ and $5x + 4y = 13$.

What is the position of their point of intersection?

27. Find the equation of the straight line perpendicular to the line with equation, $3x+5y=23$ and which passes through the point (1, 5).

28. Find the equation of the straight line that passes through the points (2, 9) and (1, 2).

29. Expand and simplify,

$$(x^2-5x+3)(3x^2+7x-4)$$

30. $4x^3+8x^2+9x+10 \div (2x+3)$

31. Simplify,

$$\frac{3x^2+2x-8}{15x^2-17x-4}$$

32. Solve,

$$x+\frac{1}{x}=\frac{26}{5}$$

33. Jamie wanted to multiply 238×479 using bar numbers (viculums) for large digits. He set out his calculation as shown on the right.

Draw circles around the places where he made mistakes.

$$\begin{array}{r} 2\ 4\ \overline{2} \\ \times\quad 5\ \overline{2}\ \overline{1} \\ \hline 1\ 1_1 8_2 0\ \overline{8}\ 2 \\ \hline 1\ 1\ 7\ 0\ 2\ 2 \end{array}$$

34. Exactly one of these equations is correct. Draw a circle round the correct one.

A $44^2+77^2=4477$

B $55^2+66^2=5566$

C $66^2+55^2=6655$

D $88^2+33^2=8833$

E $99^2+22^2=9922$

35. Which of the following is not a square?

A 1^6 B 2^5 C 3^4 D 4^3 E 5^2

36. Write the following fractions in order of size, starting with the smallest:

$$\frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447} \quad \frac{2}{227}$$

37. 50003×52467

38. $471845 \div 23$

39. Express 85 as the difference of two square numbers that are integers.

40. The ratio of Angela's age to Bill's age is 2 : 3 and that of Bill's age to Charlie's age is 4 : 7. What is the ratio of Angela's age to Charlie's age?

41. Over the course of numbering every page in a book, a mechanical stamp printed 2929 individual numbers. How many pages does the book have?

42. How many positive two-digit numbers are there whose square and cube both end in the same digit?

43. The football has 12 pentagonal panels and 20 hexagonal panels. The panels are fixed together along their edges to form joins. How many joins are there?

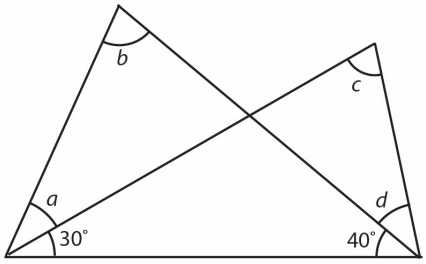
44. Pinocchio's nose is 5 cm long. Each time he tells a lie his nose doubles in length. After he has told nine lies his nose will be roughly as long as one of the following: (Draw a ring round the correct answer.)

A Domino B Tennis racket C Pool table D Tennis court E Football pitch

45. Two similar cones, A and B, have surface areas $900\,cm^2$ and $8100\,cm^2$, respectively.

If the volume of cone A is $1800\,cm^3$, what is the volume of cone B?

46. In the diagram, what is the sum of angles a, b, c and d?



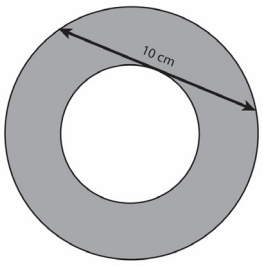
47. Roshni has the same number of brothers as she has sisters. Each one of her brothers has 50% more sisters than brothers.
How many children are in Roshni's family?

48. Five numbers are arranged in order from least to greatest.

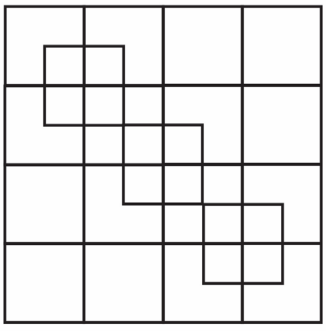
$$x \quad x^3 \quad x^4 \quad x^2 \quad x^0$$

Where does $-x^{-1}$ belong in the list above?

49. Work out the area of the shaded region, leaving your answer in terms of π. The 10 cm line is tangent to the inner circle.



50. How many squares are there?



IAVM

Sample IVMO Intermediate Solutions
Time allowed - 1 Hour

1. $4.321+43.21+432.1+4321$

$$\begin{array}{r} 4.321 \\ 43.21 \\ 432.1 \\ +4321 \\ \hline 48_1 0_1 0.631 \end{array}$$

By one more than the one before

2. Draw a ring round the number below that is divisible by 18.

834257

By elimination and retention

764522

Only the last digits

427536

869365

647381

3.

$$\begin{array}{r} 683-317 \\ \times\ 996-004 \\ \hline 680/_1268 \end{array}$$

All from 9 and the last from 10

4.

$$\begin{array}{r} 1756+756 \\ \times\ 1003+003 \\ \hline 1761/_2268 \end{array}$$

All from 9 and the last from 10

5. 7.5^2 56.25

By one more than the one before

6. 47×49

$$\begin{array}{r} 47-03 \\ \times\ 49-01 \\ \hline 2)46/03 \\ \hline 23\ 03 \end{array}$$

All from 9 and the last from 10

Proportionately

7. Divide, leaving the remainder as a whole number.

$$\begin{array}{r|l} 97 & 213/16 \\ 03 & 06 \\ & 03 \\ & 27 \\ \hline & 219/73 \end{array}$$

All from 9 and the last from 10

8. Divide,

$$\begin{array}{r|l} 113 & 259/98 \\ \bar{1}\,\bar{3} & \bar{2}\,\bar{6} \\ & \bar{3}\,\bar{9} \\ & 00 \\ \hline & 230/08 \end{array}$$

Transpose and apply

9. 43^3

64 48 36 27

96 72

15 11 2

79 5 0 7

By one more than the one before

Proportionately

10. Find the square root of the perfect square, 5776.

76

Only the last digits

*P*roduct/sum Sum/product

11. 44×404

17776

Transpose and apply

12. 3.2% of 25

$3.2\%\ \text{of}\ 25 = 25\%\ \text{of}\ 3.2 = 0.8$

Proportionately

13. 993^2 986049

14. 52.17×2.823

5 2 1 7

× 2 8 2 3

$14_4 7._3 2_4 7_6 5_1 9_2 1$

Vertically and crosswise

15. Convert $\frac{116}{125}$ to decimal.

$$\frac{116}{125} \times \frac{8}{8} = \frac{928}{1000} = 0.928$$

16. Convert the fraction, $\frac{7}{19}$, to decimal, correct to 9 decimal places.

$0._1 0\,5_1 2\,6\,3_1 1_1 5_1 7_1 8\,9\ldots$

$= 0.052631579$ (9dp)

By one more than the one before

17. Find the density of a lump of Protactinium that has a mass of 48.96 grams and a volume of $3.2\,cm^3$.

$3.^2 \overline{)\ 4_1 8._1 9\,6}$

$1\ 5.\ 3\ 0\ \text{g/cm}^3$

Vertically and crosswise

18. Convert $0.3\dot{2}\dot{5}$ to a fraction in lowest terms.

$$1000 \times 0.3\dot{2}\dot{5} = 325.\dot{2}\dot{5}$$
$$10 \times 0.3\dot{2}\dot{5} = 3.\dot{2}\dot{5}$$
$$990 \times 0.3\dot{2}\dot{5} = 322$$
$$0.3\dot{2}\dot{5} = \frac{322}{990} = \frac{161}{495}$$

By elimination and retention

Proportionately

19. Solve the simultaneous equations,

$$11x + 3y = 15$$
$$-13x + 6y = 30$$

$x = 0,\ y = 5$

When one is in ratio, the other is zero

20. Solve,

$$\frac{4x}{5} - \frac{2x}{3} = 4$$

$\frac{2x}{15} = 4,\quad x = 30$

Vertically and crosswise

21. If $6x - y = 21$ and $6y - x = 14$, what is the value of $x - y$?

$7x - 7y = 7$

$x - y = 1$

By addition and subtraction

22. Find the minimum value of the function,

$$f(x) = x^2 - 8x + 23$$

$x^2 - 8x + 23 = (x-4)^2 + 7$

Minimum $= 7$

By completion and non-completion

23. Solve by factorisation,

$$5x^2 + 17x - 12 = 0$$

$\begin{pmatrix} 5 & -3 \\ 1 & 4 \end{pmatrix}$ $(5x-3)(x+4) = 0$

$x = \frac{3}{5}$ or -4

Vertically and crosswise

When one is in ratio, the other is zero

24. Given that $(x-2)$ is a factor of,

$$f(x) = 6x^3 - 19x^2 + 9x + 10$$

find the solutions to $f(x) = 0$.

$$\begin{array}{r|l} x-2 & 6x^3 - 19x^2 + 9x + 10 \\ +2 & \quad 12x^2 - 14x - 10 \\ \hline & 6x^2 - 7x - 5 \ / \ 0 \end{array}$$

$\begin{pmatrix} 3 & -5 \\ 2 & 1 \end{pmatrix}$ $(3x-5)(2x+1)(x-2) = 0$

$x = \frac{5}{3},\ -\frac{1}{2}$ or 2

Transpose and apply

25. Find the equation of the straight line with gradient, 3, and that passes through the point (4, 2).

$mx - y = mx_1 - y_1$

$3x - y = 10$

Specific and general

26. Two lines have equations, $2x + 3y = 15$ and $5x + 4y = 13$.
What is the position of their point of intersection?

$$x = \frac{3\times13 - 4\times15}{3\times5 - 2\times4} = \frac{-21}{7} = -3$$

$$y = \frac{5\times15 - 2\times13}{7} = \frac{49}{7} = 7$$

Transpose and apply

27. Find the equation of the straight line perpendicular to the line with equation, $3x+5y=23$ and which passes through the point (1, 5).

$5x-3y=-10$

Transpose and apply

Specific and general

28. Find the equation of the straight line that passes through the points (2, 9) and (1, 2).

$7x-y=5$

Transpose and apply

Product of the means

minus product of the extremes

29. Expand and simplify,

$$(x^2-5x+3)(3x^2+7x-4)$$

x^2-5x+3

$3x^2+7x-4$

$3x^4-8x^3-30x^2+41x-12$

Vertically and crosswise

30. $4x^3+8x^2+9x+10\div(2x+3)$

$$\begin{array}{r|l} 2x+3 & 4x^3+8x^2+9x+10 \\ -3 & \quad -6x^2-3x\ -9 \\ \hline & 2x^2\ +x\ +\ 3\ /\ 1 \end{array}$$

Transpose and apply

31. Simplify,

$$\frac{3x^2+2x-8}{15x^2-17x-4}$$

$$\frac{(3x-4)(x+2)}{(3x-4)(5x+1)}=\frac{x+2}{5x+1}$$

Proportionately

32. Solve,

$$x+\frac{1}{x}=\frac{26}{5}$$

$\frac{26}{5}=5\frac{1}{5}$, $x=5$ or $\frac{1}{5}$

By inspection

33. Jamie wanted to multiply 238×479 using bar numbers (viculums) for large digits. He set out his calculation as shown on the right.

Draw a circles around the places where he made mistakes.

$$\begin{array}{r} 2\ 4\ \bar{2} \\ \times\quad 5\ \bar{2}\ \bar{1} \\ \hline 1\ 1\ 1\ 8\ 0\ \bar{8}\ 2 \\ \hline 1\ 1\ 7\ 0\ 2\ 2 \end{array}$$

34. Exactly one of these equations is correct. Draw a circle round the correct one.

Only the last digits

A $44^2+77^2=4477$

B $55^2+66^2=5566$

C $66^2+55^2=6655$

D $88^2+33^2=8833$

E $99^2+22^2=9922$

35. Which of the following is not a square?

A 1^6 B 2^5 C 3^4 D 4^3 E 5^2

By inspection

36. Write the following fractions in order of size, starting with the smallest:

$$\frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447} \quad \frac{2}{227}$$

$$\frac{4}{452} \quad \frac{4}{450} \quad \frac{4}{447} \quad \frac{4}{454}$$

$$\frac{2}{227} \quad \frac{1}{113} \quad \frac{2}{225} \quad \frac{4}{447}$$

Proportionately

37. 50003×52467

$$\begin{array}{r} 50003 + 0003 \\ \times\ 52467 + 2467 \\ \hline 2)\ 52470\ /\ 7401 \\ \hline 26235\ /\ 7401 \end{array}$$

Proportionately

All from 9 and the last from 10

38. $471845 \div 23$

$$\begin{array}{r|l} 2^3 & 4\ 7_1\ 1_1\ 8_1\ 4\ /_1 5 \\ \hline & 2\ 0\ 5\ 1\ 5/\ 0 \end{array}$$

Vertically and crosswise

39. Express 85 as the difference of two square numbers that are integers.

$$\begin{aligned} 85 &= 5 \times 17 \\ &= (11-6)(11+6) \\ &= 11^2 - 6^2 \end{aligned}$$

By addition and subtraction

40. The ratio of Angela's age to Bill's age is 2 : 3 and that of Bill's age to Charlie's age is 4 : 7. What is the ratio of Angela's age to Charlie's age?

$A : B$	$B : C$
2 : 3	4 : 7
8 : 12	12 : 21
8 : 21	

Proportionately

41. Over the course of numbering every page in a book, a mechanical stamp printed 2929 individual numbers. How many pages does the book have?

$1 \rightarrow 9 \quad = 9$ digits

$10 \rightarrow 99 \quad = 180$ digits

$100 \rightarrow 999 \quad = 2700$ digits

$9 + 180 + 2700 = 2889, \quad 2929 - 2889 = 40$

$40 \div 4 = 10. \quad 999 + 10 = 1009$ pages

Transpose and apply

42., How many positive two-digit numbers are there whose square and cube both end in the same digit?

	1	2	3	4	5	6	7	8	9	0
Last digits of squares	1	4	9	6	5	6	9	4	1	0
Last digits of cubes	1	8	7	4	5	6	3	2	9	0

Last digit 5, 9 numbers, last digit 6, 9 numbers,

last digit 1, 9 numbers, last digit 0, 9 numbers

Only the last digits Total, 36

43. The football has 12 pentagonal panels and 20 hexagonal panels. The panels are fixed together along their edges to form joins. How many joins are there?

$\frac{12 \times 5 + 20 \times 6}{2} = 90$ By inspection

44. Pinocchio's nose is 5 cm long. Each time he tells a lie his nose doubles in length. After he has told nine lies his nose will be roughly as long as one of the following: (Draw a ring round the correct answer.)

A Domino B Tennis racket C Pool table D Tennis court E Football pitch

$5 \times 2^9 = 2560\,\text{cm} = 25.6$ metres Proportionately

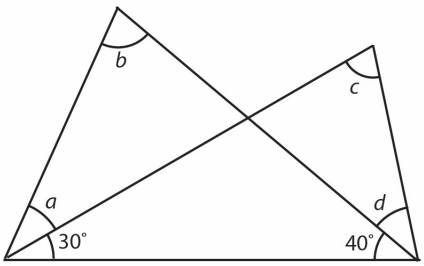
45. Two similar cones, A and B, have surface areas $900\,cm^2$ and $8100\,cm^2$, respectively.

If the volume of cone A is $1800\,cm^3$, what is the volume of cone B?

$\text{ASF} = 1:9, \quad \text{LSF} = 1:3, \quad \text{VSF} = 1:27$

$27 \times 1800 = 486{,}000\ \text{cm}^3$ Proportionately

46. In the diagram, what is the sum of angles a, b, c and d?



$a+b=110^\circ, \quad c+d=110^\circ,$

$a+b+c+d=220^\circ$

By inspection

47. Roshni has the same number of brothers as she has sisters. Each one of her brothers has 50% more sisters than brothers.
How many children are in Roshni's family?

$x+1=1.5(x-1)$ Transpose and apply

$x=5,$ Total is 11

48. Five numbers are arranged in order from least to greatest.

$$x \quad x^3 \quad x^4 \quad x^2 \quad x^0$$

Where does $-x^{-1}$ belong in the list above?

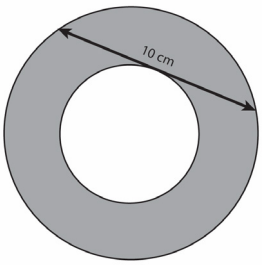
Transpose and apply

Let $x=-\frac{1}{2}$

$-\frac{1}{2} \quad -\frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{4} \quad 1$

$-\frac{1}{-\frac{1}{2}}=2,$ belongs at the end

49. Work out the area of the shaded region, leaving your answer in terms of π. The 10 cm line is tangent to the inner circle.

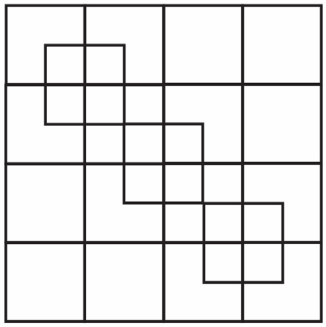


$R^2-r^2=5^2$

Difference in areas $=\pi R^2-\pi r^2=\pi(R^2-r^2)=25\pi$

Transpose and apply

50. How many squares are there?



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By elimination and retention