MYTHS AND FACTS ABOUT THE INVENTIONS BY ANCIENT SCHOLARS IN THE VEDIC MATHEMATICS ARENA

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Abstract

In order to experience the aroma of Indian Culture it is mandatory to excavate the history right from the Vedic period to the 17th century with the help of powerful tool Sanskrit. This paper mainly gives the chronology of Vedic era, Brahmanas and Sulbsutras. The works of ancient Indian scholars like Baudhyayana, Aryabhatta, Brahmagupta, Bhaskara and Madhava were pioneers of scientific thinking. The classification of the total period, from Vedic (5000BC - 3000BC), Early Classical (800BC - 300 BC) and Post Classical (1300BC -1700AD) is discussed. This paper aims to demystify the credit of inventions of the Number system, Arithmetic, Geometry and other allied branches. Mahaveera (814 AD) was the first who gave the method of Lowest Common Multiple (LCM) called Nirudha; Pingala (700BC) developed a recursive algorithm called Matra Meru to find the coefficients of different variations in the expansion of Binomials which is exactly same as the Pascal’s Triangle.

Prologue

The word Veda is a Sanskrit word derived from the root Vid which means to know. Hence Veda means a source of knowledge. There are three ordinary sources of knowledge viz: direct perception by organs, inference and comparison. Yet Veda gives us knowledge which cannot be acquired by any of the above three. Therefore Veda is a source of extraordinary knowledge (often referred to as *atindriya jnana*).

The concept of apaurusheyatva (unauthored or revealed) of Vedas is neither unscientific nor irrational dogma. It is called ‘*trikalabadhitam satyam’* such truths which existed before they were discovered and which will exist even if man would forget them in future, are called *Revelations.* Veda is a bunch of such revelations, unadulterated by man’s weaknesses.

Ancient Indians not only developed techniques to solve the mysteries of mathematics but also applied these techniques in various fields like astronomy, trade and commerce, music, grammar, navigation, construction of towns, five altars due to which India remained the most prosperous country until the 18th century.

1. Vedic Period (5000BC - 3000BC)

The three Vedas, Rigveda, Yajurveda and SamaVeda were compiled in this era and Sanskrit became the language of learned people. The decimal place value system, symbols for numerals, use of large numbers, basic knowledge of geometry and astronomy also evolved during this period.

2. Mid-Vedic period (3000 BC to 2000BC)

During the mid-Vedic period the Atharva Veda was compiled. This was the golden era of Indus-Saraswathi civilisations. Observational astronomy was also evolving. Links with foreign countries also started promoting the trade and transportation through sea routes and river routes.

3. Post-Vedic Period (2000BC to 800BC)

The Sulbasutras, Kalpasutras, Jyotisha Vedanga and Manusmrithi were compiled in this era. The Ramayana, the Mahabharatha were also compiled during this era, by 1000BC. During the Sulbhasutra period, Indians scaled new heights in arithmetic and algebraic geometry.

4. Early Classical period (800BC - 300 AD)

Takshasila became a major center of learning for Indian and foreign students around 800BC. Budhism and Jainism were born in this era. India encountered foreign invasions for the first time. Indian knowledge of mathematics and astronomy was transmitted to the Babylonians and Greeks. The entire Sanskrit grammar based on logical concepts was well stablished by 800 BC. The most famous Surya Siddhanta of Indian astronomy was first established around 800 BC. Indians kept progressing in astronomy by the use of trigonometry

5. Classical Period (300 AD to 1300AD)

Indians introduced the concepts of half chord in trigonometry that revolutionized Indian mathematical astronomy. Apart from Takshsila, Vabhi, Nalanda and Vikramasila also became major learning centers attracting students from Persia, Tibet, China. Ujjain and Kusumpura and these became centers of Indian mathematical astronomy.

Arybhata established the gravitational theory and rotation of the earth. Bramhagupta gave a general integral solution for indeterminate equations of second order. He also established the theory of second order difference interpolation. Munjala was first who gave a formula of differential calculus relating the sine and cosine functions. Bhaskaracharya also made significant contributions to the subject of differential and integral calculus.

6. Post Classical Period (1300AD to 1700 AD)

Takshsila University was destroyed during the invasion of White Huns in the 5th century. A terrorist and Barbaric invader, Bhaktiyar Khilji, attacked Nalanda and Vikramshila University in 1197AD and burnt libraries causing irreparable damage to the scientific heritage of India. Consequently, the progress in mathematics and astronomy was adversely affected in North India. South India, and particularly Kerala, became the major center of knowledge during this period. Narayana Panditha worked on permutations and combinations as well as magic squares. Madhava invented infinite series of sine, cosine, tangent and arctangent, infinite series, expansion of Pi, asymptotic expansions and Taylor series. Putumana and Nelakanta introduced planetary models.

Mahavira, The Stalwart…

Amongst the galaxy of mathematicians of India, Mahavira was one star who glorified arithmetics. The famous treatise *Ganitha Sara Sangraha* was authored by him during the reign of Manyakheta Rastrakuta, King Amoghavarsha Nripatunga. In his book Mahavir stated the 8 qualities to be possessed by a Mathematician.

1. Ability to find shortest method (*Laghukarana*)
2. Ability to have logical reasoning (*Uha*)
3. Ability to answer doubts (*Apoha*)
4. Ability to exhibit vigour in working hard (A*nalasya*)
5. Ability to grasp (*Grahana*)
6. Ability to have retention (*Dharana*)
7. Ability to find innovative solutions (*Upayi*)
8. Ability to prove and manifest (*Vyakteekarana*)

According to Mahvira "the product of the continued multiplication of the common factors of the denominators and . reducing fractions to equal denominators is also described by him.

Mahavira was first mathematician who presented a general formula for the number of permutations and combinations. Mahavira, like his anterior mathematicians, mentioned forming a rectangle from bija *m,n* (two arbitrary positive integers) instead of right triangle. Interestingly, Diophantus also expressed a similar formula of forming a right angled triangle. Mahavira also took the sides (upright, base and diagonal) as $m^2 - n^2$, $2mn$ and $m^2 + n^2$.

Mahavira gave three rules for the solution of a right triangle having a given hypotenuse or for the solution of the equation $a^2 + b^2 = c^2$ in rational integers.

Rule1: The square root of half the sum and difference of the hypotenuse and square of an optional number are the *bijas* or elements.

Rule2: The square root of the difference the squares of the hypotenuse and an optional number and the optional number will be the upright and the base.

Rule3: First form various right triangles from the *bijas* or elements, then divide the three sides of these triangles by their hypotenuses. If we multiply by the given hypotenuse, the three sides of the right triangle can be obtained. Mahavira gave four rectangles (39,52); (25,60); (33,56) and (16,63) having the same hypotenuse as 65 to demonstrate his method. Interestingly, Mahavira's method was rediscovered in Europe by Fibonacci (13th century) and Viete (16th century).

Pingala, the Genius …

Although the systematic study of poetic rhythm had been in existence since the Vedic period Pingala (brother of Panini as referred by Patanjali in his book '*Mahabhashya*') established this study exclusively in his book *ChandaShastra* in which he set out the tenents of discipline for poetic rhythms or meters in Sanskrit poetry.

Pingala pioneered the mathematical study of meters. He developed advanced mathematical concepts for explaining prosody and its characteristics. In other words, Pingala was the first to establish the relationship between the theory of poetic rhythm and combinatronics much earlier than Marin Mersenne (1588-1648 AD), a French music theorist of 17th century.

Concept of Binary Numerical System

In Sanskrit poetry, meters are based on syllabic time units (*Matras*) not on accent. Generally, a *Matra* (verse) or *Sloka* (stanza) has four quarters. Each quarter has a set of syllabic time units (*matras*). One time unit is assigned to a short syllable (Laghu) and two time units to long syllable (Guru).

Pingala classified 8 Ganas or sets of syllable triplets of long and short to define all meters of Sanskrit poetry. In this process of forming a matrix of short and long syllables, Pingala used the concept of the binary numeral system quite similar to that of Leibnitz (17th century) and Morse code. Assuming 0 or *a* for short and 1 or *b* for long, Pingala classification of Ganas (short or long syllables as bits and syllable triplets as byte) is as follows:

Sl. No.	Gana type	Binary	Algebraic	Value
1	Na	000	aaa	a^3
2	Sa	001	aab	a^2b
3	Ja	010	aba	a^2b
4	Ya	011	abb	ab^2
5	Bha	100	baa	ba^2
6	Ra	101	bab	b^2a
7	Ta	110	bba	b^2a
8	Ma	111	bbb	b^3

Even Pingala's methodology to find the lost metrical pattern of given syllables is quite similar to modern method for binary to decimal conversion.

Binomial Theorem

Pingala discussed the problem of enumeration of syllabic combinations as to how many different combinations could be produced from one of 3 syllables, 4 syllables, 5 syllables by varying short and long time units within each syllable group. In other words, he discussed combinatorics or the principles of permutations and combinations and binomial coefficients. This indicates Pingala was the first to refer the algebraic rule called binomial theorem. Truly speaking, Pingala must be given the credit for evolving a recursive algorithm for combinatorial series generation.

Pingala used two types of variables (short and long) similar to Binomial theorem. If we look at the eight Ganas referred above, the following is the arrangement for the metre having three syllables assuming a as short (Laghu) and b as long (Guru).

3 long and 1 short syllables = 1 = $1.b^3$

2 long and 1 short syllables = 3 = $1.ab^2$

1 long and 2 short syllables = 3 = $1.a^2b$

0 long and 3 short syllables =1 = $1.a^3$

This arrangement is undoubtedly the binomial expansion of $(a+b)^3$, and this method can be applied to meters having 4 or 5 syllables, and so on. Pingala applied this mathematical technique to detect the quality of the metres.

Binomial Coefficients and Pascal's triangle

Assuming the number of syllables (short and long) as n, the number of patterns follows the pattern of the binomial coefficients in the expansion of $(a + b)^n$ and the total number of patterns possible is 2^n and when $n = 4$. It becomes 16 and when $n = 5$, and so on.

Pingala developed a recursive algorithm called *Matrameru* to find the coefficents of the different variations in the expansion of $(a + b)^n$ which is exactly the same as Pascal's triangle. Moreover, Pingala's *Matrameru* method (invented around 700 BC) is much simpler than

Pascal's triangle (17th century). The method of *Matrameru* was elaborated by Halayudha (10th century), in his commentary on Chanda Shastra.

According to Pingala, the number of variations of a meter having n syllables will be obtained from representation of *Meru* (Mount or Pyramid) of *Matras* (short and long syllables).

Actually, the binomial expansion of $(a+b)^n$ for integral values of n and the case $n = 2$ was known to Indians since the Sulbasutra period (800-600BC). Hence Pascals' triangle (16th Century) should be renamed as Pingala's Triangle. Sushrutha (600BC) also applied Pingala's algorithm *Matrameru* to derrive 63 vikalpas, i.e., different combinations of six rasas = tastes (*Rasabheda Vikalpadhyaya of Sushrutha Samhita).*

Epilogue

India is a country of a rich culture and heritage. It has a long history with concrete evidences of its being a centre of knowledge far in the past.

Most of the common literature available today, such as Ramayana, Mahabharatha and Bhagavad Geetha, written at different points of time apparently dating back as far as 5000BC, were written in Sanskrit by learned people and are revered and treated as sacred Granthas by people even today. If at least any one of the self-esteemed readers of this paper nurtures the spirit of the sacred, our objective of writing this paper is fulfilled.

We would like to close this discussion with the famous quotation given by Shri Aurobindo *"Our sense of the greatness of our past must not be made a fatally hypnotizing lure to inertia; it should be rather an inspiration for greater achievement."*

Last but not least, we would like to express our profound gratitude to Sri Vedveer Arya the author of the book *Indian Contributions to Mathematics and Astronomy* which is the prime source to appear this article in this form .

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