STUDY OF APPLICATIONS OF GRAPH THEORY IN ANCIENT INDIAN SHLOKAS (SCRIPTS)

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Abstract

The Konigsberg bridge problem is a historically notable problem in mathematics. The paper published by Leonhard Euler in 1736 on seven bridges of Konigsberg was the first article in the history of graph theory. This paper deals with the investigation of shlokas in the ancient Indian scripts which discusses the concepts of graph theory and interesting unique problems related to chess.

Keywords: *Euler path, Chess, shlokas, ancient Indian scripts.*

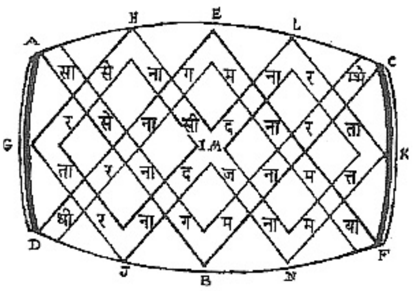
Sanskrit poetry has an incredibly vast variety of forms and structures. There is at the top the most elaborate Maha-kavya in classic style narrating a noble story element satisfying all the norms and principles prescribed by the Alankara-sastra texts. In such poetry, also known as "Uttama" (superior) kavya, the intension of a poet is to communicate the mental states of characters to the reader. There are varieties of slightly less elaborate kavyas viz, Laghu-kavya, Champu Kavya, Giti Kavyas, Mukutas, biographical poems, anthologies and stotras, etc. Among these, is a magnificent class of verse based on an adaptable play of vowels, consonants, words and sounds. The components of the poetry are, now and again, pleasantly patterned into designs (bandha), geometric figures or accompanied by pictures of usual things in life, for example, a blossom, wheel, drum, umbrella, mace, and so on. Perhaps as a result of its pictorial quality this class of poetry is known as Chitra-kavya. Unlike the Uttama class of poetry, the intention of a poet composing chitrakavya is not to convey mental states but to exhibit his skill in handling language amidst several constraints. As contrasted with Uttama kavya, chitrakavya is also referred to as Adhama kavya (inferior). The object of Chitrakavya is to inculcate admiration and wonder, to evoke amusement and pleasure, and to offer intellectual challenge.

The Chitrakavya aims to create a feeling of wonder by resorting to unusual (peculiar) management of certain meters, innovative poetic structures, designs or patterns (bandha) resembling objects (vastu) or their movements (gati) that one commonly sees in everyday life. It is 'image poetry'. Chitrakavya can be treated as architecture of poetry where the sounds of syllables (matra) and letters (akshara) take a visible form. The Chitrakavya in this way treats pictures brought to mind by the sound of the word and its meaning as separate figures (sabda –chitra and artha-chitra). It may also combine the word and the meaning into a common figure or an image (ubhaya-chitra).

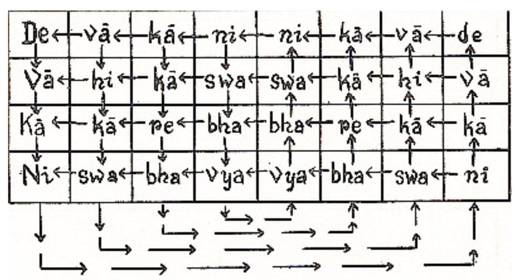
In one form of chitrakavya called Anuloma – Pratiloma (Gata – Pratyagata), the arrangement of letters is such that when read from one direction it gives one meaning and when read in the reverse direction it gives a different meaning. Sometimes the verse may give the same meaning when read either ways. Alternatively it is possible that the verse reads in one language when read forwards but in another language when read backwards. In the forms of chitrakavya called Yamaka there is a permutation of an identical set of syllabic strings where a letter or a word is repeated regularly at fixed positions in a stanza. For example,

His panicky Hispanic wife

The playful indulgence of Chitrakavya techniques were used by major poets such as Kalidasa (Raghuvamsha), Kumaradasa (Janakiharana), Bharavi (kirtarjuneeya), Magha (sishupalavadha), Ratnakara (Haravijaya), Vedanta Desikar (Padukasahasra, Yadavabhyudaya), Vadiraja (Rukmeesha Vijaya), Venkatadvari (Laxmisahasra) etc. Ramabhadracharya has done such experiments in his epic “Bhargava Raaghaveeyam”. In modern times, Ramswaroop Patak and Shataavadhani Ganesh along with Shankar are composing such stanzas even in the present day. The movement based figurative poetry (Gati Chitra) such as Gomutrika, Jalabandha anuloma-pratiloma (palindrome), sarvatobhadra, ardhabramana, turagapada patha, rathapada patha (chariot movement), gajapada patha (elephant movement) are abundantly used by many poets. Given below are the diagrammatical representations of some of the Gati chitras mentioned above.



Murajabandha -musical drum patterns

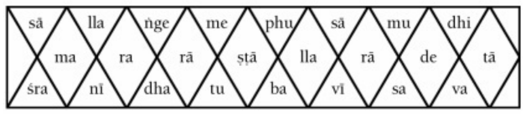


Ardha-bramana

सा	मा	या	मा	मा	या	मा	सा
मा	रा	ना	या	या	ना	रा	मा
या	ना	वा	रा	रा	वा	ना	या
मा	या	रा	मा	मा	रा	या	मा
मा	या	रा	मा	मा	रा	या	मा
या	ना	वा	रा	रा	वा	ना	या
मा	रा	ना	या	या	ना	रा	मा
सा	मा	या	मा	मा	या	मा	सा

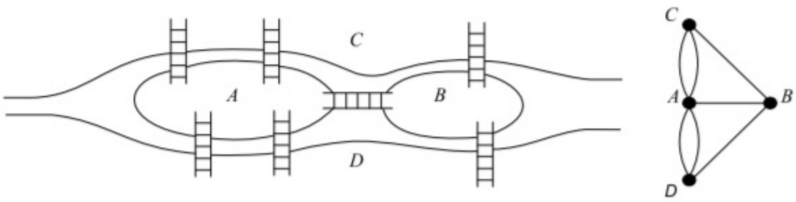
Sarvathobhadra – resembling a chess board is a type of magic square.

सा मल्लरङ्गे रामेष्टा फुल्लसारा मुदेधिता ।
श्रमनीरधरा तुष्टा बल्लवीरासदेवता ॥ ९ ॥



Gomutrika (resembles World Wide Web problem)

Graph Theory has many practical applications in various disciplines including biology, computer science, economics, informatics, linguistics, medicines and social science. The Konigsberg bridge problem is a historically notable first problem stated and solved using graph theory. In 1736, Leonhard Euler published the first article in the history of graph theory on seven bridges of Konigsberg.



The seven bridges of konigsberg and the underlying graph

This paper deals with the investigation of shlokas in the ancient Indian scripts which discusses the concepts of graph theory as applied by Sanskrit poets in composing verses and interesting unique problems related to chess.

Rudrata (9th Century) in his epic “Kavyalankara” , Bharavi in his epic “keeratarjuneeya”, Bhoja (11th Century) in his epic “Saraswati Kanthabharana”, Desikar (13th Century) “Paadukasahasram”, Kumaravyasa in his epic “Jaanakiharana” and Venkatadwari in his epic “Lakshmisahasra” have all done such experiments.

The knight’s tour problem is a mathematical problem of finding a knight’s tour around a whole chess board. It is a sequence of moves of a knight on a chessboard following certain criteria.

(i) Strict criteria: the knight has to cover all the squares and must not come to the same square twice.
(ii) Lenient criteria: the knight need not to cover all the squares and can move to the same square multiple times.

Variations of the knight's tour problem involve chessboards of different sizes than the usual 8x8 as well as irregular boards. The problem is having a knight traverse across all squares on a chessboard without visiting any square twice. The Padhuka Sahasram by Sri Vedanta Desikan presents a solution to the problem in his two verses. The first verse is written sequentially in the squares of the chessboard and the next is read along the path taken by the knight yielding the second verse as shown below.

Padhuka Sahasram

The first verse:

स्थिरागसां सदाराध्या विहताकततामता ।
सत्पादुके सरसा मा रङ्गराजपदं नय ॥

Here we have considered only half of a chess board which traces the sequence of moves made in a knight's tour.

8	स्थि	रा	ग	सां	स	दा	रा	ध्या
7	वि	ह	ता	क	त	ता	म	ता
6	स	त्पा	दु	के	स	रा	सा	मा
5	र	ङ्ग	रा	ज	प	दं	न	य
4								
3								
2								
1								
	a	b	c	d	e	f	g	h

Knight movements are shown below:

a8, c7, e8, g7, h5, f6, d5, b6, c8, e7, g8, h6, f5, d6, c8, a7 |

c6, a5, b7, d8, e6, g5, h7, f8, g6, h8, f7, e5, d7, c5, a6, b8 ||

Knight movements which lead to the second verse:

स्थिता समयराजत्पागतरा मादके गवि ।
दुरंहसां सन्नतादा साध्यातापकरासरा ॥

Kavyalankara

The following verse by Rudrata was the earliest example in which the moves will give same verse.

सेना लीलीलीना नाली लीनाना नानालीलीली ।
नालीनालीले नालीना लीलीली नानानानाली ।।

8	□□	□□	□□	□□	□□	□□	□□	□□
7	□□	□□	□□	□□	□□	□□	□□	□□
6	□□	□□	□□	□□	□□	□□	□□	□□
5	□□	□□	□□	□□	□□	□□	□□	□□
4								
3								
2								
1								
	a	b	c	d	e	f	g	h

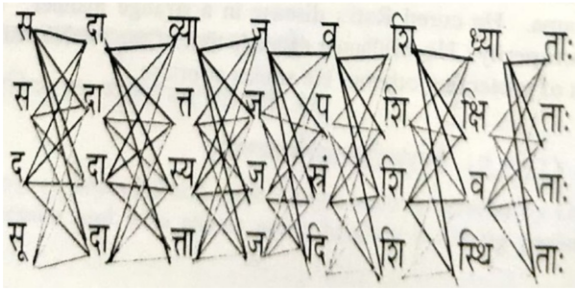
Knight movements are shown below:

a8, f5, a7, d6, c8, h6, c7, b5, h7, c6, b8, e5, b7, c5, d8, g6 |

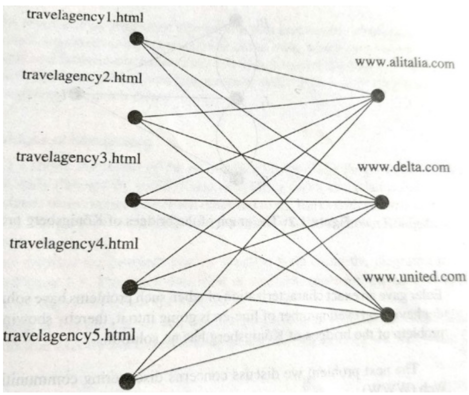
g5, h8, a6, f7, e6, f8, a5, d7, b6, g7, h5, g8, d5, e7, f6, e8 ||

Jalabandha: Every alternate (even) letter of all the four lines of the verse is the same. That is (i) 2 = 10 = 18 = 26, (ii) 4 = 12 = 20 = 28, (iii) 6 = 14 = 22 =30 and (iv) 8 = 16 = 24 = 32 as shown below.

सदाव्याजवशिधायाताः सदात्तजपशिक्षिताः ।
ददास्यजस्त्रं शिवताः सूदात्ताजदिशि स्थिताः ।।



Each block of the above Jalabandha is a bipartite graph (relationships among objects) which can be modeled as a World Wide Web graph where the Web pages are represented by dots or vertices and the hyperlinks between them are represented by lines or edges in the graph. The graph shown below represents a Web community which is also known as a complete bipartite graph in graph theory.



World Wide Web community graph

We have considered only half of a chess board and we have presented an algorithm which traces the sequence of moves made in a knight's tour. The usual approach to write an algorithm to generate a knight’s tour is to try out all the possible solutions until we get a solution which satisfies all our constraints. The more efficient approach is called backtracking. This starts from one square and keeps going to the next move until a dead end is reached. At this point it backtracks to the previous move and tries another possibility. Using this we can get all possible knight’s tour sequence moves of a knight in an 8*4 chessboard. In this, one is our solution according to which another "shloka" is generated from the original one.

Pseudo code /algorithm:

1. Arrays which define details of all 8 possible movements of a knight.
2. Function to check if (x, y) is valid chessboard coordinates.
3. Print a solution if all the positions are filled or else go to the next move using recursion and then backtrack back when a dead-end is reached.
4. Initializes the visited array and calls the Knight Tour () function starting from the 1st Position.

Conclusion

This paper makes a novel attempt to bring together the disciplines of Graph theory and Sanskrit poetry/poetics by analyzing certain types of pattern poetry (Chitra Kavya) using the principles of Graph theory. Though Sanskrit poets, are far removed from us on a time-scale, the verses composed by them exhibit some similarity with concepts dealt with by Graph theory. In addition to the various applications of Graph theory in modern disciplines, we could surmise that it is also useful in composing particular forms of constrained poetry. This does not however imply that the Sanskrit poets were aware of graph theory but only that the Chitra kavya verses that they composed are compatible with Graph theory and can be studied in the latter's background.

Acknowledgements

The authors would like to thank to Dr. Shankar Rajaraman, Ph.D. Scholar, Indian Institute of Science, Bangalore for his guidance and contribution for this work. The authors express their gratitude to the parent institution for the support and encouragement.

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