# Contribution of Indian Mathematics to the World

Daya Tiwari

Abstract

The wealth of ancient Indian Mathematical Science has contributed a lot to the world civilization. Ironically, modern India seems to have forgotten its rich ancient heritage treasure preserved in the vast Sanskrit literature. The Science of Mathematics with all its branches such as Arithmetic, Algebra, Geometry and Trigonometry, etc., was so well developed in ancient India that many modern scholars find to their dismay that some of the European discoveries have already been discovered long ago. It is the need of hour to integrate this valuable treasure of Indian scientific knowledge with that of modern science. India has the unique distinction of combining the three concepts - 1. Decimal system, 2. place value and 3. a computational zero (shunya). The origin of Geometry is from Shulvashutra.
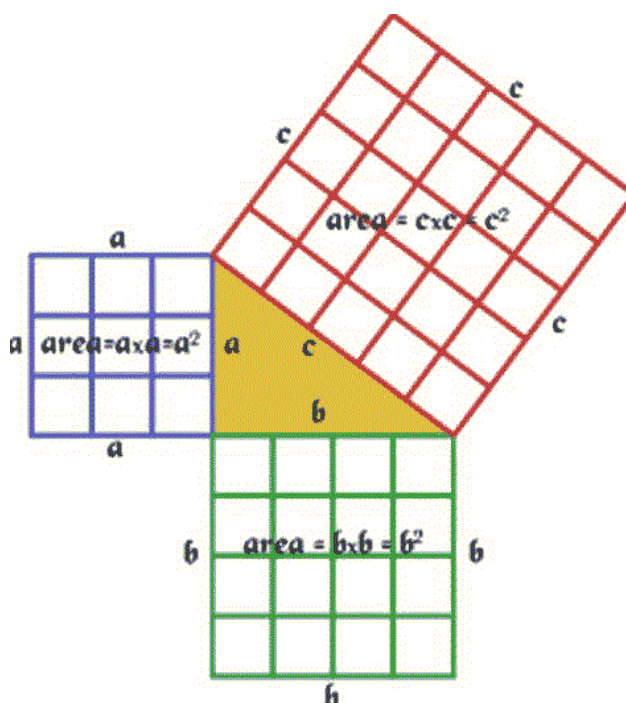
The equation $c^2 = a^2 + b^2$, which is known as Pythogoras theorem, was firstly given by Baudhayana (800 BC) before Pyhthagoras (580-500 BC). This is basically shulva theorem (BaudhayanaShulvashutra, 1-48). There is clear reference of numbers in the Vedas. In the Rigveda, we find one and twenty numbers (Rigveda; 7.18.11), ten and hundred numbers (Rigveda; 6.47, 22-24). In the Ayurveda, the powers of 10 from 10 to the power 10 and 10 to the power 12 are listed (Yajurveda17.2). Arithmetic progression (AP) i.e. series of multiple of 4 is mentioned in the Yajurveda (18.25). Numbers with the description of Ayut (10,000) and Nyarbud (ten crorers) are in the Atharvaveda (8.8.7). Valmiki's Ramayana (kanda 6th sarga28, shloka 33-43) mentions not only the strength of Rama's numbers but also establishes a relation between the numbers.

In Aryabhatiyam (Ganitpada, 10), Aryabhatta 1st (476 AD) has given the value of π (pi) 3.1416, which is correct to four decimal places and it has been universally accepted.

This research paper is aimed at determining the mathematical facts with examples and proofs from various ancient Sanskrit texts. There shall also be a special focus on the blend of ancient Indian mathematics and universal modern mathematics.

The importance of mathematics has been highlighted in the **VedangaJyotisha**(1400 B.C.)of Lagadha:

Yathashikhamayuranamnaganammanayoyatha,

Tadavatvedangashastranamganitam murdhanisthitam.[1]

i.e. Like the crests on the heads of peacocks, like the gems on the heads of the cobras, Mathematics is at the top of the Vedanga Sastras.

Indian Contribution to Numeral System, Decimal System, Place Value, Zero and Infinity:

We find clear description of Numeral System, Decimal System, Place Value and Zero in our Vedas, Brahman Granthasand the magnum opus-the Ramayanaand the Mahabharataand other texts. The Vedic names of numerals being used today indicates that the decimal system assigning symbols 1-9 and the concept of place value and zero fully evolved during Vedic period.

There is clear reference of numbers in the Vedas. In the Rigveda we find one and twenty

numbers in this mantra–

Ekam chayovinshati chashravasya……Esham.[2]

Such as 'gaining of ten treasures, ten horses, ten golden pieces (roundish mats), ten chariots with horses and hundred cows[3]are mentioned in the mantras of Rigveda. In the Yajurveda, the powers of 10 from 10° to 10^{12} are listed.[4] [Ek (10°=1), 10^1 (Dasa), 10^2 (Shat) 10^3 (Sahsra-thousand), 10^4 (Ayuta-Ten thousand) ……10^8 (Nyarbud- One hundred millions), 10^9 (Samudra- One thousand millions), 10^{10} (Madhya- A ten thousand millions), 10^{11} (Anta- A hundred thousand millions) and 10^{12} (Parardha- a million million ora billion)]. Taittiriya Samhita[5] also mentions this list of numerals.

Numbers with the description of 100 and 1000 Ayut (means 10,000) and Nyarbud (meaning ten crores) are mentioned in the following mantra of theAtharvaveda-

BrihatejalambrihatIndraShurasahasrarghasyashataviryasya,

Ten shatamsahasramayutamnyarbudamjaghanashakrodasyunamabhidhaya senaya.[6]

 Valmiki Ramayan mentions not only the strength of Rama's but also establishes a relation between the numbers-

Shatamshatasahasranamkotimahurmanishinah,

Mahabalavritonityammahabala parakaramah.[7]

In **Aryabhatiyam** (499 A.D.) it is also mentioned that-

Ekamdasha cha………….dashagunam syat.[8]

Arithmetic Progression (AP) i.e. series of multiples of 4 that is also table of [4, 8, 12, 16……..48] is mentioned in the Yajurveda in this mantra–

Chatasrashchameshtauchmeashtauchame…

ashtachatvarinshatchameyajnenakalpatam.[9]

Four basic mathematical operations (+, -, x, ÷) have been used in the Vedas. In the Rigveda, we find number 21 as an addition of 20+1, 107 as 100+7 etc.[10] Rigveda[11] mentions 8 - 7 =1. Maitrayani Samhita[12] mentions 12x3=36, 12x2=24. "Dwidha", "Tridha" and "Ansha", etc., words of the Vedas indicate the operation of the division. In Taittiriya Samhita[13], the division of 1000 has been mentioned into three parts.

Fractions are referred to for the first time in the Rigveda[14]. As these fractions are called 1/4(pada), 1/2 (ardha), 3/4 (tri-pada) etc. Maitrayani Samhita[15] shows the fractions 1/16 (kala), 1/12 (kushtha), 1/8 (sapha) and 1/4 (pada). The decimal based system having compound numbers like 11(Ekadasha-Eka+dasha), 21(Ekavimshati-Eka+vimshati), 27(Saptavimshati-Sapta+vimshati), etc., also indicate the place value system.

We find in the symbolic sign of zero (shunya) in Rigveda and "rupeshunyam" for prastara in Pingalachhandashastra (200 BC). The idea of expressing all quantities by 1-9 figures and every new series of powers of 10 like dasha (2 digit series), shata (3 digit series), sahasra (4 digit series), etc., and Dvidashati (vimshati=2x10), panchdashati (panchashat=5x10), etc., symbolize the concept of zero as integral part of Vedic numeral system.

The concept of infinity found in the Yajurvedaand Brihadaranyakopanishad[16]

Yatrayampurushomriyatekimenamnajahatitinametyanantamvai,

Namanantavishve deva anantamevasa ten lokamjayati.

and found at another place as –

Yadevate ……pratishthananta …………nananushishyahareteti.

The Peace invocation of Ishopanishad also highlights the importance of infinity. The invisible is the infinite, the visible too is the infinite. From the infinite, the visible universe of infinite extension has come out. The infinite remains the same, even though the infinite universe has come out of it.

Om purnamadahpurnamidampurnatpurnamudachyate,

Purnasyapurnamadayapurnamewavashishyate.

Contribution in the field of geometry

Vedic people were fully acquainted and have the knowledge of applied geometry as well. The origin of Geometry is from Shulbasutra. The Vedic people used to make sacrificial altars in definite prescribed shapes and sizes using special types of bricks. The KatyayanaShulbasutrabelongs to the Shuklayajurveda, the Baudhayana and Apastambasutra belong to the Krishna-Yajurveda. The KatyayanaShulbasutrabegins with the statement *"rajjusamasamvyakhyamah"*. Shulbasutra of Vedanga taught the important rules to construct triangles, rectangles, squares, parallelograms and circles by explaining their properties.

The equation $AC^2 = AB^2 + BC^2$ which is known as Pythagoras Theorem was firstly given by Baudhayana (800B.C.) before Pythagoras (580-500 B.C.).This is basically Shulba Theorem-

Dirghachaturasrasyakshnyarajjuhparshvamani,

Tiryangmani cha yatprithakbhutekurutastadubhayam karoti.[17]

i.e. the areas produced separately by the length and the breadth of a rectangle together equal the area (of the square) produced by the diagonals.

$$AC^2 = AB^2 + BC^2$$
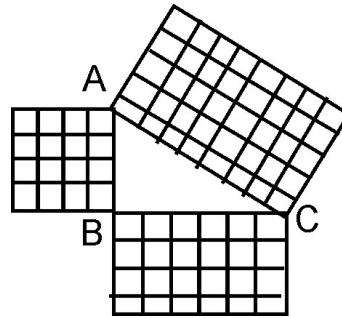
Contribution in the field of trigonometry

Trigonometry was an important gift of ancient mathematicians to the mathematical would. They used *'jya'* and *'Kojya* which are now *'Sine'* and *'Cosine'* in European trigonometry.

Aryabhata I (476-540 A.D.) gave the lines of angle between Zero to Ninety (0-90). This was used by astrologers to decide the actual place of planets. Aryabhata gives the method of calculating the dimension of a shadow cast by an object placed in the cone of the light coming out of a lamp or a source of illumination and, by applying the rule of three in the geometry of triangles, he gives a simple rule in respect to these shadows. This forms the basis of calculating eclipses[18].

Aryabhata was the first to mention the most accurate value of π (Pi) which is correct to four decimal places. According to him –

Chaturadhikamshatamashtagunamdwashashtistathasahasranam,

Ayutdvayavishkambhasyasanno vrittaparinahah[19].

i.e., If we add four (4) to one hundred (100), multiply it by eight (8) and add to sixty two thousand (62000) to that number, the result is approximately the circumference of a circle whose diameter is twenty thousand.

$$\pi \text{ (Pi)} = \frac{\textbf{\textit{Cirumference}}}{\textbf{\textit{Diameter}}} = \frac{62,832}{20,000} = 3.1416.$$

This value of π has been universally accepted and widely applauded by the whole mathematicians even today.

Contribution in the field of algebra

The ancient Indian nomenclaturefor the science of algebra is *bijaganita, bija* means elements or analysis and *ganita* means the science of calculation. Brahmagupta (598-678 A.D.) calls algebra *kuttaka-ganita,* meaning pulveriser, which refers to a branch of science of algebra dealing with the subject of indeterminate equations. He also gave rules for solving various types of quadratic equations.

Bhaskara II, in his *Bijaganita,* showed his strength in his ability to handle problems of indeterminate equations. He gives both algebraic as well as geometrical solutions for these problems.

The invention of the computational Zero gave an impetus unmatched to all branches of knowledge. Indian contribution to fractions (indirectly to rational numbers) is to be recognized with great respect. The ancient Indian mathematicians made use of fractions, although, in their symbols, they did not use a fraction bar. Brahmagupta gave rules for working with fractions. These rules are quite like the rules we use today. From India, the fractions and rules for working with them spread into the Arab world. It was translated into Arabic by the famous Arab mathematician Al-Kowarizmi (825 AD). From the Arab world, the Indian work spread to Italy and the West.

By making perfect square, the method to solve a *quadratic equation (ax^2 +bx+ c=0)* was given by Sridhara (750 AD).

Chaturahatavargasamaihrupiah pakshadvayamgunayet,

Avyaktavargarupairyuktaupakshautato mulam[20].

viz. multiply by the four times of the coefficient of the square of unknown quantity in both fields.

By solving the equation

$$ax^2 + bx + c = 0$$

Multiply both sides by 4a, $\qquad 4a^2x^2 + 4abx + 4ac = 0$

Subtract 4ac from both sides, $\qquad 4a^2x^2 + 4abx = -4ac$

Add b^2 to both sides,
$$4a^2x^2 + 4abx + b^2 = -4ac + b^2$$

Since, $(m + n)^2 = m^2 + 2mn + n^2$

Complete the square on the left side, $(2ax + b)^2 = b^2 - 4ac = D$

Take square roots, $2ax + b = \pm\sqrt{D}, \quad 2ax = -b \pm \sqrt{D}$

and, divide by 2a,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This formula of Sridharais is used in modern mathematics for solving quadratic equations.

Contribution in the field of permutations and combinations

Mahavira (814-880 AD), the greatest Jaina mathematician of Karnataka, discovered the formula of permutations and combinations (^nC_r). Contemporary scholars used this in music, making of poetry and in the field of medicine. He authored the famous book *Ganitasarasamgrah*. As example-

Ekadyekottaratahpadamurdhwardharyatahkramotkramasah,

Sthapyapratilomadhanampratilomadhanenbhajitamsaram.

i.e. Start with 1 and increasing by 1 let numbers going up to the given number of things be written down in regular and inverse order in the upper and lower rows respectively. If the product of numbers in the upper row taken from right to left be divided by the product of numbers in the lower row taken from right to left, the number of combinations is obtained.

$$N_{c_r} = \frac{n(n-1)(n-2)\ldots\ldots\ldots(n-r+1)}{1.2.3\ldots\ldots\ldots r} = \frac{n!}{r!(n-r)}$$

This formula was later rediscovered from Indian sources by the French mathematician Herigone in 1634.

Mahavirawas was the first mathematician to discover the LCM (Lowest Common Multiple) method and named it as "Niruddha". The process of reducing fractions to equal denominators is also mentioned by him. This method was followed by the European countries in the 15th Century.

Other contributions in the field of mathematics

Bhaskara (1114 AD)[21] gave the theory of 'Surface Area' and 'Volume of a Sphere' etc.

(Surface area of a sphere $=4\pi r^2$, Volume of a sphere $= 4/3\pi r^3$)

Srinivasa Ramanujan (1887-1920) worked on the contemporary mathematics of his time but his approach was typical. His most mathematical research was intuitive and very little of it comes by having axioms and deducing theorems from axioms. He has given unique formulae as *Squaring the circle, Nested square roots, Partitions of natural numbers, some approximations for π,* and *Magic squares*. Prof. Julian Huxley called him "the greatest mathematician of the century" and the renowned historian of mathematics, E.T. Bell, called him "a gift from heaven".

The French mathematician Pierro –Simon Laplace (1749-1827) had said, "*it is India that gave us the ingenious method of expressing all numbers by means of 10 symbols, each symbol receiving a value of position as well as an absolute value. The idea escaped the genius of Archimedes and Apollonius*".

Albert Einstein has marked the Indian contribution,*"We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made."*

Indian Scientist Dr.A.P.J. Abdul Kalam has written about the importance of ancient Sanskrit literature- "*Ancient Sanskrit literature is a store-house of Scientific principles and methodology. The work of our ancient scholars should be thoroughly examined and where possible integrated with modern science".*(Ignited Minds, P.87)

Thus, the Indian Mathematics has contributed a lot to the world which is universally accepted even today. Now the book *"Vedic Mathematics"* of Bharati Krishna Tirth which has sixteen Sutras (formulae) and their sub Sutras (Corollaries) from the Vedas is used in India for short cut methods of solving multiplication, division, factorization, sum and differences, etc.

References

1. Vedangajyotisha ;4

2. Rigveda ; 7.18.11

3. Ibid ;6.47,22-24

4. Yajurveda;17.2

5. TaittiriyaSamhita; 7.2.11-20

6. Atharvaveda ; 8.8.7

7. Valmiki Ramayana; Kanda VI, sarga 28, verses 33-43

8. Aryabhatiyam ; Ganitapada, verse 2

9. Yajurveda; 18.25

10. Rigveda; 7.18.11, 1.45.2, 1.34.11

11. Ibid; 10.72.8, 10.72.9

12. MaitrayaniSamhita; 1.10.8

13. TaittiriyaSamhita; 7.1.5

14. Rigveda; 10.90.4

15. MaitrayaniSamhita; 3.7.7

16. Brihadaranyakopanishad; 3.2.12, 4.1.5

17. BaudhayanaSulbasutra; I.48

18. Aryabhatiyam; Ganitapada, 14-16

19. Ibid; Ganitapada, 10

20. Bhaskara II quoted in "Avyaktavarga Equation" of Bijaganita

21. Lilavati; Kshetravyavaharaprakrana; 165-6

Bibliography

1. Acharya, Sudumna, 2006, *Ganita-ShastrakeVikaskiBharatiyaParampara*, MotilalBanarasiDass, Delhi.
2. Archak, K.B.&Micheal(ed.), 2007, *Science, History, Philosophy and Literature in Sanskrit Classics*, SundeepPrakashan, Delhi.
3. Arya, vedveer, 2014, *Indian Contribution to Mathematics & Astronomy,* Aryabhata Publications, Hyderabad.
4. Bag, A.K., 1979, *Mathematics in Ancient and Medieval India*, Chaukhamba Orientalia, Varanasi.
5. Banerjee,M; Goswami, B.(ed.), 1994, *Science and Technology in Ancient India,* Sanskrit Pustak Bhandar, Calcutta.
6. Chaubey,B.B, (ed.), 2009, *Rigveda volume I &II,*IGNCA, New Delhi.
7. Dwivedi, Sudhakar, 1927, *Bijaganitam of Bhaskara II*, Banaras Sanskrit series
8. Jha, Lashanlal, 2008, *Lilavati* , ChaukhambaVidyabhavan, Varanasi.
9. Kalam, APJ, Abdul, 2002, *Ignited Minds*, Penguin Books India, Panchsheel Park, New Delhi.
10. Kashyapa , R.L.(ed.),2004, *TattirīysaŠhitā*, Sri Aurobindo Kapali Shastri Institute of Vedic culture, Bangaluru.
11. Mishra,K.K.(ed.), 2003, *Sanskrit Vānmayamein Vijñana ka tihsasa* , NCERT, New Delhi.
12. Prakash, Satya, 1965, *Founders of Sciences in Ancient India,* The Research Institute of Ancient Scientific Studies; New Delhi.
13. Rangacharya, M., (ed), 1912, *Ganitasarasangraha*, Madras.
14. Rao, Balachandra, S., 1994, *Indian Mathematics and Astronomy,* Jnana Deepa Publications, Banglore.

15. *Sh*ā*nkarbh*ā*sh*ya, Samvat 2061, *Brihadāranyakopanishad* , Gita Press, Gorakhpur.

16. Satavalekara,S.D.(ed), 1942, *Maitrayani Samhita*, Aundh.

17. Sharma, Punita (comm.), 2008, *Vedangajyotish*, Nag Publication Delhi.

18. Sharma, Satyadev (comm.), 2008, *Aryabhatiyam*, Chaukhamba Surbharati Prakashan, Varanasi.

19. Sharma, Ram Swarup (ed.), 1966, *Brahmasphuta Siddhant of Brahmagupta*, Indian Institute of Astronomical and Sanskrit Research, New Delhi.

20. Sen,S.N.; Bag,A.k., 1983, *Sulbsutras*, Indian National Science Academy, New Delhi.

21. Shastri, Ramkrishna *(trans.)*, 1999, *Yajurveda(shukl*ayajurveda*),* Chawkhambhavidyabhawan,Varanasi.

22. Tirtha, Bharati Krishna, 2005, *Vedic Mathematics*, Motilal Banarasidass, Delhi.

23. Vasishtha, Shivram Sharma (ed.),1992, *"*Shrimad*valmikiramayana",* Chawkhambhavidyabhawan,Varanasi.

24. Whitney, W.D. *(trans.)*, 1996, *Atharvaveda*, MotilalBanarasiDass,Delhi.