REVERSE OSCULATION PROCESS FOR EVEN DIVISORS

Mrs. Geeta Ghormade & Dr.Anant Vyawahare

Abstract

1. This paper deals with the reverse osculation process giving stress to even divisors, so that, even though the dividend is not divisible by the divisor, one can get the quotient and remainder. The Vedic sutras and sub-sutras used in this paper are,

1. Ekadhikena Purvena (By one more than the previous one)

2. Eknyunena Purvena (By one less than the previous one)

3. Veshtanam (By Osculation)

4. Sopantyadwayamantyam (Ultimate and twice the penultimate)

5. Anurupyena (Proportionately)

6. Sishyate Seshasanjna (The remainder remains constant)

2. For confirming the divisibility of divisors, especially the higher prime numbers as a factor, Swami Bharti Krishna Tirthji developed a simple and attractive osculation process using Vedic formula *Veshtanam* (by osculation) which shows whether a given number (dividend) is divisible by a given divisor (a prime number). The process is carried out from the ones place to higher places (right to left) of the dividend. The introductory method checks for divisibility for divisors ending in 1, 3, 7 or 9.

3. Instead of working from right to left of dividend, the divisibility can also be checked by osculating from left to right (highest place to the ones place). This process is called *Reverse Osculation*. It is similar to the division process, but instead of dividing by a divisor, we divide by its osculator which is a very small number compared with the divisor and this will obtain both the quotient and the remainder.

If the divisor is an even number, then we can also use reverse osculation.

1. The Reverse Osculation Process with odd divisors

In case when divisors end with 1, 3, 7 or 9, the osculator is calculated using the Vedic formula Ekadhikena Purvena and Eknyunena Purvena.

a) For divisors ending with 9, add 1 and cancel zero to get its osculator. Thus for 29, add 1 to get 30 and cancel the zero of 30 to get 3 as positive osculator *p* for 29. Likewise, for divisors 19, 29, 39, 49 …… , the positive osculators, *p,* are 2, 3, 4, 5 …respectively.

b) For divisors ending with 1, subtract 1 and cancel zero to get its osculator. Thus, for 41 subtract 1 to get 40 and cancel zero to get 4 as the negative osculator, *q*. Likewise, for divisors 21, 31, 41, 51…, negative osculators, *q,* are 2, 3, 4, 5… respectively.

c) For divisors ending with 3, first multiply divisor by 3 so that it ends with 9, and then find *p*. Thus for the divisor 23, first 23 x 3 = 69 so that *p* for 69 is 7. Likewise, for divisors 13, 23, 43…., positive osculators, *p,* are 4, 7, 13…. respectively.

d) For divisors ending with 7, first multiply divisor by 3 so that it ends with 1 and then find *q*. Thus, for divisor 17, first 17 x 3 = 51 so that *q* for 51. Thus for divisors 7, 17, 27…., negative osculators, *q,* are 2, 5, 8….respectively.

Each divisor can have both a positive and negative osculator, and the sum of these is always the divisor itself! For example, for the divisor 17, as 17 x 3 = 51, its *q* is 5 and as 17 x 7 = 119, its *p* is 12. The divisor 17 = *12 + 5 = (p + q)*. The same is true for any odd divisor.

2. Reverse Osculation Process for Odd Divisors

The examples to be the solved by the reverse osculation process have 3 rows. The first row is of dividend, second of osculated sum (for p) or osculated difference (for q), and the third row is of quotient digits. Here, we will be dividing by the osculator (p or q) in terms of quotient (Qt) and remainder (R). The quotient digit is written in the third row below the leftmost dividend digit, and the remainder is prefixed to the next order (second from left) dividend digit. When osculator is positive (p) , the first quotient digit is added to the number formed by prefixing dividend digit and written below second dividend digit (from left) in second row. If the osculator is negative (q), the quotient digit is subtracted from that number and written below second dividend digit in the second row.

The obtained sum (for *p*) or difference (for *q*) is osculated by *p* or *q* in terms of quotient and remainder. Write this Qt ahead of first Qt in third row & prefix R to next order dividend digit. Continue the addition/subtraction of Qt digit followed by its osculation till unit place of dividend is reached. If the final sum below unit place in the second row is zero, divisor, or its multiple, then the dividend is divisible by the given divisor. The number formed in third row is Qt, whereas final osculated sum in second row is R.

If the final sum is divisor itself, as it is equivalent to 1 Qt, add 1 to Qt so that R is zero. If this sum is more than divisor convert into proper Qt & R, and rearrange for the final Qt & R. if this sum is less than divisor, it is R. Thus for divisor 29, if final sum/difference is 17, it is R. if it is 29, it is 1 Qt & zero R, whereas if it is 34, it is 1 Qt + 5 R.

Example 1 Find Qt & R for 62381 ÷59 (p for 59 = 6)

Row 1 (Dividend)	6	${}_{0}2$	${}_{3}3$	${}_{3}8$	${}_{1}1$	
Row 2 (Osc. sum)		3	33	43	18	Final sum 18 is R
Row 3 (Qt digits)	1	0	5	7		

Qt for 59 is 1057

Write the dividend in Row 1 keeping space between digits, and proceed stepwise as follows:

Step 1: (6 ÷osc. 6) = 1 Qt + 0 R prefixed to 2

Step 2 : (02 + 1) ÷ osc. 6 = 0 Qt + 3 R prefixed to 3

Step 3: (33 + 0) ÷ osc. 6 = 5 Qt + 3 R prefixed to 8

Step 4: (38 + 5) ÷ osc. 6 = 7 Qt + 1 R prefixed to 1

Step 5: (11 + 7) = 18 final R

∴ 62381 ÷59 = 1057 Qt + 18 R

All steps from 1 to 5 can be carried out orally to get Qt & R.

Example 2 Find Qt & R for 88293 ÷ 41

Row 1 (Dividend)	8	${}_{0}8$	${}_{2}2$	${}_{1}9$	${}_{2}3$	
Row 2 (Osc. sum)		6	21	14	20	Final sum 20 is R
Row 3 (Qt digits)	2	1	5	3		

Qt for 41 is 2153

Step 1: (8÷osc. 4) = 2 Qt + 0 R prefixed to 8

Step 2 : (08 - 2) ÷ osc. 4 = 2 Qt + 2 R prefixed to 2

Step 3: (22 - 1) ÷ osc. 4 = 5 Qt + 1 R prefixed to 9

Step 4: (19 - 5) ÷ osc. 4 = 3 Qt + 2 R prefixed to 3

Step 5: (23 - 3) = 20…….final R

$\therefore 882931 \div 41 = 2153$ Qt + 20 R

Let us solve following example directly.

Example 3 Find Qt & R for 25802 $\div$ 23

As 23 x 3 = 69, p for 23 is 7

$$\begin{array}{ccccc|l} 2 & {}_{2}5 & {}_{4}8 & {}_{2}0 & {}_{6}2 & \\ \hline & 25 & 51 & 27 & 65 & \text{R for 69 is 65} \\ \hline 0 & 3 & 7 & 3 & & \\ \hline \end{array}$$

As osc. for 23 is 7, Qt & R obtained is for 69 which is 3 times divisor 23

$\therefore$ Qt for 23 = Qt for 69 x 3 = 373 x 3 = 1119 using *Proportionately*

$\therefore$ R for 23 = 65 (23 x 2) + 19 = 2 Qt + 19 R

$\therefore$ 25803 $\div$ 23 = (1119 + 2) Qt + 19 R = 1121 Qt + 1

In this way, by reverse osculation process, using p and q for given divisors, Qt and R can be easily found for odd divisors.

3. Reverse Osculation Process for Even Divisors

With slight modification Qt and R can be found out for even divisors (ending with 2, 4, 6 and 8). To find the osculator in this case, twice of Ekadhikena Purvena and twice of Eknyunena Purvena Vedic formulae are used. For the osculation process for odd divisors, the Qt digit was added (for p) or subtracted (for q) from the prefixed dividend, whereas in the case of even ending divisors, *twice* the Qt digit is added (for a positive osculator) or subtracted (for a negative osculator) from the prefixed dividend digit. The rest of the procedure is the same as carried out for odd divisor discussed above. Thus, here the Vedic formula, Sopanta Dwayamantyam (the ultimate and twice the penultimate) comes into the picture.

Let us see how to find positive and negative osculators for divisors ending with 2, 4, 6 or 8.

Divisors ending with 8:

Add 2 to divisor so that it ends with zero. Drop zero to get positive osculator. Thus, for 28, add 2 (28 + 2= 30) and drop zero of 30 to get 3, a positive osculator for 28. As we are adding

2, let us denote $\dot{p}$ (p dot) for such positive osculators to distinguish from p (for odd ending divisors). Thus $\dot{p}$ for 18, 28, 38, 48…. ending with 8 is 2, 3, 4, 5, respectively.

Divisors ending with 2:

Subtract 2 from divisor so that it ends with zero. Drop zero to get the negative osculator. Thus for 42, subtract 2 (42 – 2 = 40) and drop the zero of 40 to get 4 as nagative osculator. As we are subtracting 2, let us denote $\dot{q}$ (q dot) for such negative osculators to distinguish from q (for odd ending divisors) Thus, $\dot{q}$ for divisors 12, 22, 32, 42….. ending with 2 is 1, 2, 3, 4, respectively.

Divisors ending with 4:

a) Multiply by 2 so that it ends with 8. Add 2 and drop zero to get its positive osculator. Thus for 24, as 24 x 2 = 48, adding 2 (48 + 2 = 50) and dropping zero for 50 gives 5 as $\dot{p}$ for 24 Thus for divisors 14, 24, 34, 44, ending with 4, $\dot{p}$ will be 3, 5, 7, 9, respectively.

b) Multiply by 3 so that it ends with 2. Subtract 2 and drop the zero to get its nagative osculator. Thus for 24, as 24 x 3 = 72 subtracting 2 (72 – 2= 70) and dropping zero of 70 gives 7 as $\dot{q}$ for 24. Thus for divisors 14, 24, 34, 44, ending with 4, $\dot{q}$ will be 4, 7, 10, 13, respectively.

Divisors ending with 6:

a) Multiply by 2 so that it ends with 2, and then as above find the nagativeve osculator $\dot{q}$. Thus for 16, as 16 x 2 = 32, subtracting 2 and dropping zero for 30 will give $\dot{q}$ for 16 as 3. Thus for divisors 16, 26,36, 46, ending with 6, $\dot{q}$ will be 3, 5, 7, 9, respectively.

b) Multiply by 3 so that it ends with 8, and then find positive osculator as above. Thus for 16, as 16 x 3 = 48, adding 2 and dropping zero of 50 will give $\dot{p}$ as 5. Thus for divisors 16, 26, 36, 46…. ending with 6, $\dot{p}$ will be 5, 8, 11, 14, respectively. Each even divisor also can have both positive and negative osculators $\dot{p}$ and $\dot{q}$, and the relation between $\dot{p}$, $\dot{q}$ and the divisor D is $2(\dot{p} + \dot{q}) = D$.

For example, for $D = 16$, $\dot{p} = 5$, and $\dot{q} = 3$ which shows 2(5 + 3) = 16. The same is true with other even divisors. Hence if $\dot{p}$ is known, $\dot{q}$ can be found out, and vice versa. Normally for practical purposes, the lesser of $\dot{p}$ or $\dot{q}$ is preferred for the osculation process. Now, let us solve one example each with $\dot{p}$ and $\dot{q}$ step wise to find Qt & R by reverse osculation.

Example 4 Find Qt & R for 6719 $\div$ 28

$\dot{p}$ for 28 is 3 (so divide by 3 and add twice the Qt digit to prefixed number)

$$\begin{array}{cccc} 6 & {}_{0}7 & {}_{2}1 & {}_{0}9 \\ \hline & 11 & 27 & 27 \\ \hline 2 & 3 & 9 & \\ \hline \end{array} \quad \text{Final sum, R is 27}$$

Qt for 28 = 239

Step 1: $(6 \div 3)$ = 2 Qt + 0 R prefixed to 7

Step 2 : (07+2 x 2) $\div$ osc. 3 = 3 Qt + 2 R prefixed to 1

Step 3: (21+2 x 3) $\div$ osc. 3 = 3 Q + 2 R prefixed to 9

Step 4: (09 +2 x 9) =27 R

All these steps can be orally carried out

$\therefore$ $6719 \div 28$ = 239 Qt + 27 R

Example 5 Find Qt & R for 5544 $\div$ 42

$\dot{p}$ for 42 is4 (so divide by & subtract twice the Qt digit to prefixed number)

$$\begin{array}{cccc} 5 & {}_{1}5 & {}_{1}4 & {}_{0}4 \\ \hline & 13 & 8 & 0 \\ \hline 1 & 3 & 2 & \\ \hline \end{array} \quad \text{Final sum, R is 0}$$

Qt for 42 = 132

Step 1: $(5 \div 4)$ = 1 Qt + 1 R prefixed to 5

Step 2 : (15 - 2 x 1) $\div$ osc. 4= 3 Qt + 1 R prefixed to 4

Step 3: (14 - 2 x 3) $\div$ osc. 4 = 2 Q + 0 R prefixed to 4

Step 4: (04 - 2 x 2) = 0 R

All these steps can be orally carried out.

$\therefore 5544 \div 42$ = 132 Qt + 0 R (Hence 5544 is divisible by 42)

Now let us solve some examples directly.

Example 6 Find Qt & R for 26576 $\div$ 48

$\dot{p}$ for 48 is 5

$$\begin{array}{cccc}
26 & {}_{1}5 & {}_{0}7 & {}_{2}6 \\
\hline
 & 25 & 17 & 32 \\
\hline
5 & 5 & 3 & \\
\hline
\end{array}$$

$\therefore$ 265764 $\div$ 48 = 553 Qt + 32 R

Example 7 Find Qt and R for 52056 $\div$ 82

$\dot{q}$ for 82 is 8

$$\begin{array}{cccc}
52 & {}_{4}0 & {}_{4}5 & {}_{7}6 \\
\hline
 & 28 & 39 & 68 \\
\hline
6 & 3 & 4 & \\
\hline
\end{array}$$

$\therefore$ 52056 $\div$ 82 = 634 Qt + 68 R

We have seen that these divisors can have both positive and negative osculators and we can test for divisibility using either.

The following example is solved using $\dot{p}$, as well as $\dot{q}$.

Example 8 Find Qt and R for 8398 $\div$ 26

As 26 x 2 = 52 $\dot{q}$ for 26 is 5. Also, 26 x 3 = 78, $\dot{p}$ for 26 is 8.

a) with q = 5,

$$\begin{array}{cccc}
8 & {}_{3}3 & {}_{1}9 & {}_{2}8 \\
\hline
 & 31 & 7 & 26 \\
\hline
1 & 6 & 1 & \\
\hline
\end{array}$$

$\therefore$ Qt for 26 = Qt for 52 x 2, 161 x 2 = 322

R for the divisor 52 is 26. Since the original divisor is 26 this gives 1Qt + 0 R

$\therefore$ Qt for 26 = 322 + 1 = 323 and R = 0. Hence 8398 is divisible by 26.

b) with $\dot{p} = 8$,

8	$_{0}3$	$_{5}9$	$_{3}8$
	5	59	52
1	0	7	

R for 78 = 52 and Qt for 78 = 107

$\therefore$ Qt for 26 = Qt for 78 x 3 =107 x 3 = 321

R for 26 = R for 78 adjusted for 26 = 52 which is 2 Qt + 0 R

$\therefore$ Qt for 26 = 321 +2 = 323 & R = 0. Hence 8398 is divisible by 26.

Thus by both $\dot{p}$ & $\dot{q}$ we get same result.

In this way, choosing proper $\dot{p}$ *or* $\dot{q}$, one can determine the divisibility (R = 0) or both Qt and R when divisors end with 2, 4, 6 or 8.

In the similar fashion, divisors with a series of nines ending with 8 or a series of zeros ending with 2 can be converted into proper *px* or *qx* by grouping *x* number of digits from the right of dividend – in the similar fashion as carried for *px* and *qx*. The only change being adding twice the Qt group (for *px*) or subtracting twice the Qt group (for *qx*) from the prefixed group until the ones place group result is reached.

Example 9 Find Qt and R for $8503488 \div 1998$

As 1998 + 2 = 2000, cancelling three zeroes, $\dot{p}3 = 2$ and hence we should group 3 digits from the right of dividend.

8	$_{0}503$	$_{1}488$
	511	1998
4	255	

R = 1998 = 1 Qt + 0 R

$\therefore$ 8503488 $\div$ 1998 = 4256 Qt + 0 R. (divisible)

Example 10 Find Qt and R for $762470833 \div 30002$

As 30002 - 2 = 30000, cancelling 4 zeroes, $\dot{q}4 = 3$ and hence the dividend is grouped by four digits from the right.

$$\begin{array}{lll} 7 & {}_{1}6247 & {}_{1}0833 \\ \hline & 16243 & 5 \\ \hline 2 & 5414 & \\ \hline \end{array}$$

$\therefore$ 762470833 $\div$ 30002 = 25414 Qt + 5 R.

Example 11 Find Qt and R for 348624818 $\div$ 7998

For 7998, $\dot{p}3 = 8$

$$\begin{array}{lll} 348 & {}_{4}624 & {}_{1}818 \\ \hline & 4710 & 7994 \\ \hline 43 & 588 & \\ \hline \end{array}$$

$\therefore$ 348624818 $\div$ 799 = 43588 Qt + 7994 R

Conclusion

1. In this way, the reverse osculation process can be used to determine divisibility of even ended (2, 4, 6 and 8) divisors that will give both Qt and R simultaneously. Also the process can be further extended to divisors with a series of nines ending with 8, and a series of zeros ending with 2.

2. Definitely, this reverse osculation process would save time, space, and energy to obtain quotients and divisors simultaneously.

3. The steps of calculation are, in fact, identical to those of the Vedic method of Straight division for two-digit divisors, including the use of vinculum digits in the flag. Straight division actually becomes very much easier when the second digit of the divisor is a 1 or 9. It is also quite easy when the final digit is 2 or 8. However the important point is that, by using the Proportionately rule, divisors ending with 3, 4, 6 or 7 can easily be altered so that they end with 1, 2, 8 or 9.

References

1) Vedic Mathematics by Swamiji Bharati Krishna Tirth

2) Discover Vedic Mathematics by Kenneth Williams

3) Magical World of Mathematics by V.G.Unkalkar