VEDIC MATHS IN EDUCATION

NUMBER SYSTEM, MODULAR ARITHMETIC AND VEDIC SUTRAS

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The importance of modular arithmetic is on my mind and it struck me that number system, modular arithmetic and Vedic sutras are related and all follow rhythmic multiple patterns.

Cardinal names like one, two… are just adjectives of distinction of group size, however the ordinal faculty is more powerful because it acts on the objects and the groups and places, holds the members in an order. I can always order and name the group size at the same time, and this is called counting or numbering. Therefore ordering is prior to naming most of the time and in that sense ordinal names precede cardinal names.

Scientists and commercialists, and dancers and artisans prefer to use ordinal names, poets and those less scrupulous utilise cardinal names.

The stimulus response arc is part of a cycle and thus periodicity is involved in the ordinal faculty. We have a connection directly to the rotational motion in space as the source for all the innate sensibility to order.

Again the rotation is modelled by the modular arithmetic systems, and these are used as the building blocks for aggregation structures like the decimal, binary, hexadecimal and sexagesimal systems.

The numbering response is the same as the singing response found in most animals, particularly birds and mammals. This singing response when responding to the rhythmical quality of an environmental stimulus is akin to counting out the beat, making up a song with words, free flowing alliterative associative responding.

Sanskrit poetry consists of two kinds of syllables, short and long.

Long syllables are Stressed (guru) and short syllables are Unstressed (laghu).

Fibonacci and Rythms

Write down 1 and 2.

Each subsequent number is the sum of the previous two

The *n*th number we write down is the number of rhythms on *n* beats.

1,2,3,5,8,13,21,34 ,… - Hemachandra numbers (c.1050 AD)

The 8th number in this sequence is 34. So there are 34 rhythms of 8 beats consisting of long and short syllables

Example, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55,….

In the west, these numbers are called Fibonacci numbers after the Italian mathematician Fibonacci. They appear in nature's art, for example, fruitlets of a pineapple or the flowering of an artichoke. Fibonacci numbers were discovered by scholars in ancient India (1700 years before Fibonacci) when they were analyzing rhythms in Sanskrit poetry!

Computers understand machine language. Every letter, symbol etc. that we write in the instructions given to computer gets converted into machine language. This machine language comprises of numbers. In order to understand the language used by computers and other digital systems it is crucial to have a better understanding of number system.

The relation between number and modular arithmetic

Theorem: For any integer *n*, the Fibonacci sequence modulo *n* is a periodic sequence.

Examples

{Fn} = 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ….

{Fn mod 3} = 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0 …

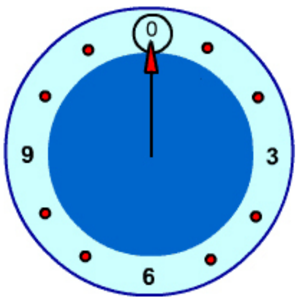
{Fn mod 7} = 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0,1,1,2,...

We can visualize the modulo operator by using circles.

We write 0 at the top of a circle and continuing clockwise writing integers 1, 2, ... up to one less than the modulus.

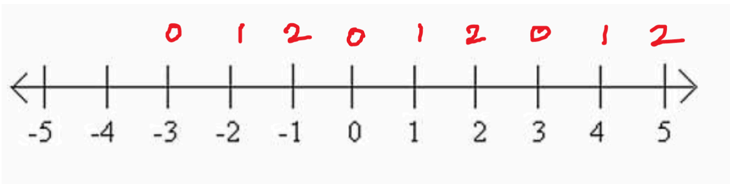
Mod12={1, 2, 3 ……………………………..,11}

For example, a clock with the 12 replaced by a 0 would be the circle for a modulus of 12.



All these stuck me while I was dealing with problems on numbers including exponentials. It is worth mentioning Fermat’s little theorem on exponents relating modulo operandi. Fermat”s little theorem states: If *p* is a prime number and *p* does not divide *a* , then $a^{p-1} \equiv 1$ (mod*p*).

For almost all its history, the study of modular arithmetic has been driven purely by its inherent beauty and by human curiosity. But in one of those strange pieces of coincidence which often characterize the advance of human knowledge, in the last half century modular arithmetic has found important applications in the real world.” Today, the theory of modular arithmetic (e.g., Reed-Solomon error correcting codes) is the basis for the way DVDs store or satellites transmit large amounts of data without corrupting it. Moreover, the cryptographic codes which keep, for example, our banking transactions secure are also closely connected with the theory of modular arithmetic. You can also visualize the usual arithmetic as operating on points strung out along the “number line.”



Mod3= {0, 1, 3}

Set out below are some examples showing how periodicity relates to numbers and modular arithmetic

Unit digits defining Periodicity:

Digit ‘d’	d^2	d^3	d^4	d^5
1	1	1	1	1
2	4	8	6	2
3	9	7	1	3
4	6	4	6	4
5	5	5	5	5
6	6	6	6	6
7	9	3	1	7
8	4	2	6	3
9	1	9	1	9

Digits	Period
0,1,5,6	1
2,3,7,8	4
4,9	2

Question1

What is the unit digit of 7^5 ?

$7^5 = 7 * 7^4 = 7 * 1 (\text{mod}10) =$ -----7 Application of modular arithmetic or

$7^5 = 7\,(7^4) = 7\,((7^2)^2) = 7\,(49^2) = 7 * (2401) = \ldots\ldots\ldots 7$

Vedic method - Nikhilam or Urdhva-Tiryagbyham (for squaring 49)

or

Since the periodicity of 7 is 4 Divide 5 by 4 the remainder is 1, hence 7^1, the unit digit is 7

Question2

What are the last four digits of 7^{128}?

$7^{128} = (7^4)^{32} = (2401)^{32} = (2400+1)^{32} = 1 + 32*2400 + {}^{32}C_2\,(2400)^2$
$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{mod}10000$

$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6801(\text{mod}10000)$

or

7^128= (7^4)32 = (2401)^32=(2400+1)^32= 1 + 32*2400+$^{32}C_2$ (2400)2
+……………………………..

= 1 + 32* 2400 +………………………………….

=1+76800+……………………………= ……………………….6801

Ignore the mod function because we are interested in last 4 digits. The binomial expansion from third term onward will have 10000 as last 4 digits.

Multiplications can be done using any of the Vedic methods

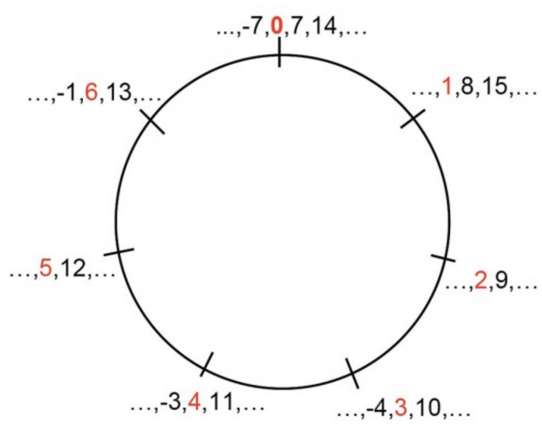
Question3

What is the unit digit of 17 ^17^17?

$(17 \wedge 17)^{17} = (17 \wedge 16 * 17)^{17} = (17 * 289^{8})^{17} = (17^{17})\,(289)^{8*17} = 17 * (289)^8\,(289)^{136} = 17\,(289)^{144}$

Unit digit of 289^{144} is 1 and multiplied by 17, so the unit digit is 7.

Below is a representation of Modulo 7



Question4

What are the last 2 digits of $2005+2005^2+2005^3+$……………………………2005^{2005}?

$2005+(2005^2+$………………………………..$+2005^{2004})$ squaring concept of 2005 it always ends in 25

5+2004* 25 (mod100) =………………..05

or

5+2004 time 25 which gives unit digit as 05 2005^n=…………25 as last 2 digits except n=1

Question5

Find the remainder when $3^{100,000}$ is divided by 53. (Application of Fermat little theorem)

Here 53 is prime number, 53 is not divisible by 3 ,

$3^{53-1} \equiv 1 \pmod{53}$

$3^{52} \equiv 1 \pmod{53}$

100,000 divided by 52, Quotient is 1923 and remainder is 4

$(3^{52})^{1923} \equiv 1^{1923} \pmod{53}$

$(3^{52})^{1923} * 3^4 \equiv 1^{1923} * 3^4 \pmod{53}$

$3^{100,000} \equiv 81 \pmod{53} \equiv 28 \pmod{53}$

Question6

Find the remainder when product of 44 and 113 is divided by 12.

44 = 3*12 + 8

113 =(3*12+8) (9.12+5)

= (3*9) 12^2+ (3*5+8*9) 12 + (8*5)

Highlighted are multiples of 12. Therefore, 40 divided by 12 gives Q3 and r=4

Or 44 ≡ 8 (mod12), 113≡5(mod12)

Therefore, 44*113=8*4(mod12) ≡4(mod12)

Mod or Modulus is defines as , if A divided by B gives quotient Q and remainder R, then AMod B =R.

The rotation is modelled by the modular arithmetic systems, and these are used as the building blocks for aggregation structures like the decimal, sexagesimal, and binary systems.

Using the sutra *All from 9 last from 10* we can turn subtraction into addition by a kind of tens complement. For example, 267 minus 198 is 69. Instead of subtracting 198, this can be done by adding the complement of 198 and then subtracting 1000.

$\begin{array}{r} 1000 \\ -198 \\ \hline 892 \end{array}$ and then $\begin{array}{r} 267 \\ +802 \\ \hline 1069 \end{array}$ from which 1000 is subtracted to give 69.

This little rearrangement makes subtraction from left to right straightforward and gives you the power of the modular arithmetic. The 1 is outside the range of the subtrahends in the clock cycle 0 to 999.

The same problem in Hexadecimal system:

$$\begin{array}{r} 10\,B \\ -C\,6 \\ \hline 4\,5 \end{array} \qquad \begin{array}{r} 15\,16 \\ -C\ \ 6 \\ \hline 3\ \ A \end{array} \qquad \begin{array}{r} 10\,B \\ +3\ \ A \\ \hline 14\ \ 5 \end{array}$$

Here, the 1 is outside the range of the subtrahends in the clock cycle 0 to 16.

This can be generalised to any modular arithmetic, so for the sexagesimal system it is *All from 59 and the last from 60*, or *All from 15 and the last from 16* for the hexadecimal system, or *All from 7 and the last from 8* for the octal system, and finally *All from 1 and the last from 2* for the binary system.

These last 3 modular arithmetics are used in binary computing and it is satisfying to see this modern connection to Vedic mathematics.

Modular arithmetic also supports the meaning of shunya as "full" rather than empty, or if you wish it has both meanings, but my point is that Brahmagupta has a different conception of arithmetic when he says 1-1= shunya and introduces “fortunate” and “unfortunate" numbers. He is advising on modular arithmetic showing how every cipher has its complement.

Also the modular mathematics is aligned to Vedic mathematics.

Numbers (evident from Vedic period), rhythm, and modular arithmetic all relate to one another and with this no machine language is possible. Inclusion of Vedic mathematics in curriculum will be in line with modern advancement in mathematics education.