ADVAITA AND THE SUTRAS OF VEDIC MATHEMATICS

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Introduction

As Shankaracharya, Swami Sri Jagadguru Bharati Krishna Tirtha expounded the ancient philosophy of unity known as Advaita – Non-dualism. Avaita is a philosophy originally expressed in Vedic texts, Brahma Sutras and Upanishads. The greatest exponent of Advaita was the first Shankara (Adi-Shankara). His commentaries on the Brahma Sutras express this teaching. He also showed how the disparate approaches to Vedic teachings, found within Buddhism and Jainism, could be unified within Advaita.

Adi-Shankara expressed the essence of Advaita in four statements known as the Mahavakyas, These are,

Aham brahmasmi – I am Brahman

Tat tvam asi – Thou art that

Ayam atma brahma- This Self is Brahman

Prajnanam brahma – Brahman is knowledge

These quintessential teachings set out the philosophy of Advaita – Non-dualism, and state that the individual Self (Atman) is non-different from the Universal Self (Brahman) and that the Self is pure consciousness. Of philosophies, it is reckoned to be very ancient and certainly predates various spritual paths derived from the Vedas such as Hinduism, Jainism and Buddhism.

The highly respected tradition of the Shankaracaryas has evolved and developed from the words and works of the Adi-Shankara. Shankaracaryas are regarded as the highest authorities on Vedic teachings and Bharati Krishna Tirtha was no exception. At the same time he found a great love of mathematics and through his studies of ancient texts, together with deep penetration and understanding of human nature and the human psyche, drew together sixteen sutras describing all mathematical operations. It is reasonable to suppose that his approach to mathematics reflects aspects of Advaita philosophy. This paper investigates some of the connections between Advaita philosophy and six of the sutras of Vedic mathematics.

Unity within Mathematics

Since unity is the main theme of Advaita and features as what may be argued as the starting point of mathematics it is worthwhile looking at some of the properties of the number one.

a) The simple act of counting, 1, 2, 3, and so on, is nothing but a repeated application of 1.

b) Each number is separated by one.

c) Each number is a single entity and is therefore a unity in itself. Human consciousness naturally cognizes unity. For example, a tree with thousands of leaves is seen as a single entity.

d) The number one is the identity element for multiplication and division. When a number is multiplied or divided by one it returns to itself and is unchanged.

e) The number one can be seen as indivisible by considering what happens when 1 is divided by 2. Firstly, in primary school we learn that $1 \div 2 = 0$ remainder 1. This shows that 1 has not been divided because it is there unchanged at the end of the process. Secondly, with decimal arithmetic we set out the division process as,

$$2\overline{)1.0}\,0.$$

The first step is as before, 1 divided by 2 is 0 remainder 1. Again, no division has taken place but the 1 remaining is now placed in the tenths place and turned into 10. $10 \div 2 = 5$.

$$2\overline{)1.0}\,0.5$$

So in this case the 1 has not been divided except by "pretending" it is something else, namely 10.

The third way we try and divide 1 by 2 is to write, $1 \div 2 = \frac{1}{2}$

Of course, this is a real cheat because it says that 1 divided by 2 is 1 divided by 2 and so doesn't answer the question. However, we give this a name and a concept, namely, "half". These illustrations show that the number one cannot be divided except by pretending that it can. The play of number begins with this sort of pretence.

> *In the beginning there was mere being, one without a second. That being thought, "Would that I were many! I will create".*
>
> Chhandogya Upanishad Book VI

Nikhilam Navatascaraman Dasatah – All from 9 and the last from 10

1, 10, 100, and so on, are all expressions of unity and this sutra relates any number to the nearest power of 10 above. For example, the complement of 842 is 158, obtained by subtracting all the digits from 9 but the last from 10. In Vedic mathematics, complements are frequently used to make an arithmetic problem easier. For example, $98 \times 7 = 700 - 14 = 686$

Another example, 782×996

$$\begin{array}{r} 782 - 218 \\ \times\ 996 - 004 \\ \hline 778 \quad 872 \end{array}$$

All from nine and the last from ten brings out the natural interplay between the numbers and their deficiencies or complements from the base, which makes calculations light and enjoyable.

In a lecture on Advaita philosophy, Sri Bharati Krishna Tirtha said,

> *The natural state is happiness. This is not to say that unhappiness is never experienced, but that it exists only as the result of some specific cause. Remove the cause, like a toothache, and then unity and happiness are naturally available…*

He then mentions the Nikhilam sutra,

> *In Vedic mathematics, the sutra All from nine and the last from ten, similarly shows that unity is always present. For example, 897 looks like a large and awkward number, but following the sutra we find, 9 – 8 = 1, 9 – 9 = 0, 10 – 7 = 3. Adding 897 + 103 = 1000 uncovers the unity behind the complicated appearance.*
>
> Vedic Metaphysics

Puranapuranabhyam – By Completion and Non-Completion

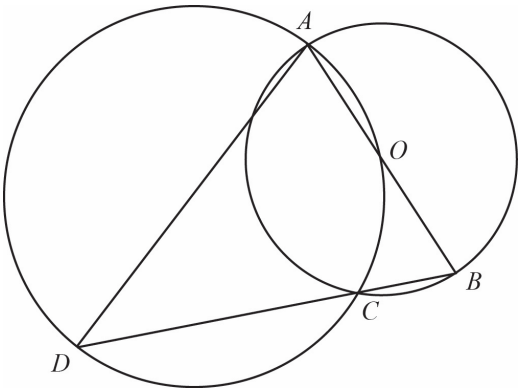
Completion is a state a state of unity and in mathematics as well as astronomy dealing with the complete and non-complete are common features.

An example can be seen in solving a quadratic by completing the square, such as $x^2 - 6x - 27 = 0$.

$$\begin{aligned} x^2 - 6x + 9 - 27 &= 0 + 9 \\ (x-3)^2 - 27 &= 9 \\ (x-3)^2 &= 36 \\ x - 3 &= \pm 6 \\ x &= 9, -3 \end{aligned}$$

More generally, with mathematical problems of all kinds we seek a resolution or completion.

For example, in this problem the question is, given the larger circle passes through the center of the smaller circle at O, does $\angle DAB = \angle DBA$?



Having been presented with the problem, while there is insufficient knowledge, the question is unresolved and incomplete. The inquisitive student will naturally seek to find the answer and thereby gain a sense of completeness.

> *He who has found Him, seeks no more; the riddle is solved; desire gone, he is at peace. Having approached from everywhere that which is everywhere, whole, he passes into the whole…. As rivers lose name and shape in the sea, wise men lose name and shape in God, glittering beyond all distance."*
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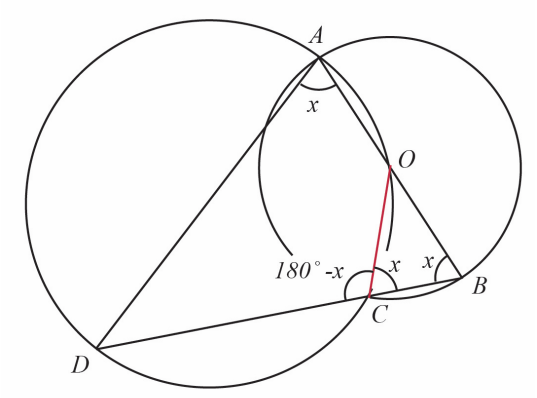
> Mundaka Upanishad, Book 3



To solve this question, draw a line from O to C. In the larger circle, $DAOC$ is a cyclic quadrilateral within the larger circle. As a consequence, if $\angle DAB = x$ then $\angle DCO = 180 - x$ and $\angle OCB = x$. Since OC and OB are radii, triangle OCB is isosceles and so $\angle DBA = x$.

The geometry of this problem is simple but until the line OC is formed it remains difficult. The line OC is the key to bringing the problem to completion.

In astronomy, completion and non-completion are essential ideas in connection with attempting to resolve cycles of orbits. No two astronomical cycles are commensurate. This means there is always a remainder when comparing two cycles. This is particularly relevant to constructing calendars. The most widely used calendar is the Gregorian named after Pope Gregory XIII 435 years ago. The time taken for one complete revolution of the Earth around the Sun is not exactly 365 days but is approximately 365.24219 days. Due to the remainder, 0.24219 days, a leap day is inserted every four years and as a further correction 3 leap days are omitted every 400 years.

A prominent ancient Indian calendar, described in the Rik Recension of Jyotisha Vedanga, sets out a lunar-solar calendar based on a five-year cycle with a leap half-month every 2.5 years named after the previous half-month. Imagine your holiday month comes to an end and then starts all over. Very generous!

In the arithmetic of decimal fractions we find they are either complete (non-recurring) or incomplete (recurring).

$$\frac{1}{40} = \frac{1}{2 \times 2 \times 10} = 0.025$$

$$\frac{1}{7} = 0.\dot{1}42857\dot{}$$

Where the factors of the denominator consist entirely of 2s and 5s the decimal equivalent is non-recurring, that is, complete. For each factor of 2, 5 or 10, there is one digit. For example 1/40 has three decimal digits. Where the denominator has other rime factors the decimal equivalent is recurring and non-complete.

Paravartya Yojayet – Transpose and Apply

In Tirthji's book this sutra appears more frequently than any other. He translates it variously as *Transpose and Adjust* and *Transpose and Apply*. But what do the words literally mean? The prefix, *para*, has a multitude of meanings such as away, along, off, back. *Vartya* is from the root *Vrit*, meaning to turn. So Paravartya literally means, *turn back* or *transpose*. The verb *Yojayet* is from the root, *Yuj*. It means unite or join. The word yoga comes from the same root. In terms of Advaita, yoga ultimately means the uniting of the individual Self and universal Self. So one literal meaning of the sutra is *Turn back and unite*. This is a reflection of what yoga is all about.

But what has yojayet, joining, got to do with apply? When working with simple arithmetic calculations there are innumerable occasions when numbers unite. For example, with 2 x 3 = 6, the 2 and the 3 merge together to form the product, 6. The idea here is that, by means of an operation, two numbers become united.

Tirthaji's very first example for this sutra is with algebraic division.

$$\begin{array}{r|l} x-2 & 12x^2-8x-32 \\ +2 & \quad 24x \quad 32 \\ \hline & 12x+16 \;/\; 0 \end{array}$$

The second term of the divisor, -2, is transposed to +2, and this becomes the multiplier applied to every quotient term. So it merges with each quotient term through multiplication.

Having shown how to use the sutra for polynomial division he then proceeds to show the identical method for numerical division for cases where the divisor is a little more than a power of 10. His first example is 1234 ÷ 112.

$$\begin{array}{rrr|l} 1 & 1 & 2 & 1 \quad 2 \quad 3 \quad 4 \\ & -1 & -2 & \quad -1 \; -2 \\ & & & \quad\quad -1 \; -2 \\ \hline & & & 1 \quad 1 \;/\; 0 \quad 2 \end{array}$$

In this case the digits of 12 are transposed to -1 and -2. These are then used as the multipliers on each quotient digit.

In chapter 11, Tirthaji translates the sutra as *Transpose and Adjust* and proclaims it as the rule for transposing terms from one side of an equation to the other with the inevitable change of sign. This gives a slightly different meaning to the word Yojayet. The root Yuj can also mean to fix for example in harnessing oxen with a yoke, and so adjust is entirely in keeping.

$$\begin{aligned} 3x+2 &= 5x-7 \\ 9 &= 2x \\ x &= \tfrac{9}{3} \end{aligned}$$

Vyashti Samashti – Specific and General

Vyashti literally means *The being differentiated* or separated and so comes to mean *individual* or *particular*. Samashti means, *The being altogether as one*, and so means

universal. In Advaita, Vyashti Samashti indicates the non-difference between the individual Self (Atman) and the universal Self (Brahman).

> *Ayam atma brahma - This individual Self is the universal Self*
>
> Mandookya Upanishad

Elsewhere we read of the universal self reflected in each individual.

> *God thought: "I would be many; I will procreate. And in the heat of his meditation created everything; creating everything He entered into everything; entering into everything He took shape yet remained shapeless; took limits yet remained limitless; made his home yet remained homeless; created knowledge and ignorance; reality, unreality; became everything.*
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> Taittireeya Upanishad, Book 2

In his book, Shankaracarya Tirtha only mentions the sutra once, in Chapter 19, in connection with solving a special type of biquadratic equation. He does not give a translation but intimates that it has to do with finding an average. When considering what an average is its easy to appreciate that its an individual number that in some way represents or reflects something of a whole group of numbers. Of course there are several types of average and the sutra is an umbrella to all of them. In finding a simple mean the Samashti is the total of all the numbers and by dividing by the number of individuals we arrive at an individual number that reflects the whole.

This idea of an individual reflecting the whole goes very much further than just finding a mean. We see it in self-similarity, in fractals within Chaos theory, in proportion, in recurrence relations, and wherever there is pattern where the whole is reflected in each part. The idea or recognition or perception of an individual part reflecting the whole goes beyond mathematics. It is a key aspect of nature and also of how we perceive the world around us.

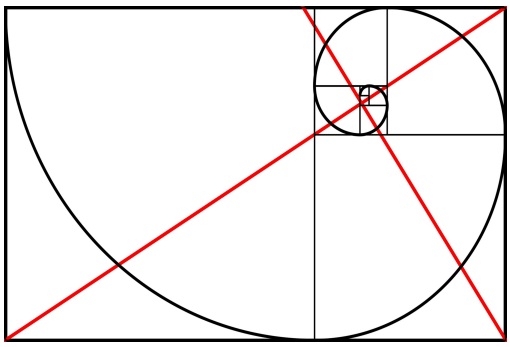
Consider tulips. Each tulip is recognisable as tulip because there is a common structure or design amongst all tulips. At the same time each tulip flower is an individual and demonstrably different from every other. So each individual has a universal element and also an individual element. The same is true for any species of any creature. This is recognised and demonstrated even at the molecular level of DNA in which there are common features of genetic code in all living creatures and, obviously, individual characteristics as well. Between humans, for example, 99.9% of DNA is identical. Compared with other creatures the proportion is lower but still very high, for example, mice 85% and cattle 80%.

We also find the same theme running through philosophy, civilisations, culture, nations, tribes, families, art, invention and design.

The great western philosopher Socrates examined the ideal state as a reflection of the individual and vice versa. He considered the attributes of the state as a reflection of the individuals living within that state. In all civilisations, nations, tribes, ethnic groups and families there are common characteristics as well as individual characteristics.

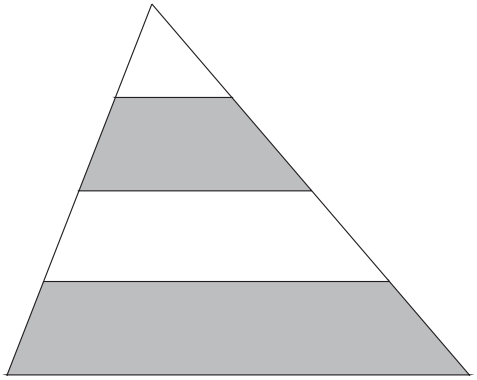
Other examples can be found in design. For instance, an interior designer perceives each individual artifact in relation to the whole room so that furnishings, colour, and so on, have a harmonious relationship, not only with each other, but also in relation to the whole.

In mathematics this sutra has fundamental applications. Amongst the foremost is the principle of self-similarity. A common example is the golden rectangle in which, with each cut of a square, the remaining shape is the same as the whole.

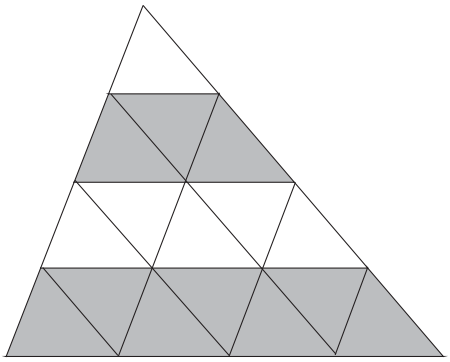


Anurupyena - Proportionately

Closely related to Specific and General is *Proportionately* and one of the obvious applications is in that of similarity. The following problem illustrates the use of both sutras. The triangle has two of sides divided into quarters and the lines joining form bands as shown. What fraction of the triangle is shaded?



This is solved by drawing a grid of triangles each of which is a replica of the whole. There are 16 small triangles, 10 of which are shaded, and so 5/8 is the answer.



Similarity is often used in design giving a sense of harmony. The example below is of a pagoda in Indonesia.

Sunyam Samyasumuccaye – When the total is the same, it is zero

This intriguing sutra has a profound meaning. It speaks of equality. A simple interpretation is, when two totals, or things, are the same then there is no difference. Equality is central to mathematics (an many other areas of life) and this sutra appears to give a definition.

An example from algebra is when a common factor is present.

$$5(2x-3)+\frac{2(2x-3)}{7}=19(2x-3)$$

$$\therefore\ 2x-3\ =\ 0, \qquad x=1.5$$

The theme of equality runs throughout Advaita and is based on recognition of sameness.

> *Uddalaka said, "By knowing a lump of clay you know all things made of clay; they differ from one another as it were in language and in name, having no reality but their clay".*
>
> Chhandogya Upanishad, Book VI

The Vedic teaching uses an analogy of the sun shining in pots filled with water. The reflection of the sun can be seen in each pot. When looking at the pots there appear to be many suns but, in reality, there is just one sun.

Conclusion

This paper has aimed at describing how some of the sutras of Vedic mathematics reflect aspects of the teachings of Advaita. Sri Tirthaji's principal aim was happiness as is reflected in his definition of Dharma,

> *The sum total of all means necessary for speedily making and permanently keeping all the people, individually as well as collectively superlatively comfortable, prosperous, happy, and joyous in all respects (including the physical, mental, intellectual, educational, economic, social, political, psychic, spiritual etc. ad infinitum).*
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> Vedic Mathematics

He described Vedic Mathematics as "mathematics without tears" and if students and readers find this to be the case then we may say that his job is well done.

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