Sample IVMO Senior
Time allowed - 1 Hour

1. 984756×999997

2. Draw a circle round the number below that is divisible by 45.

814250
728515
637425
128590
439365

3. 52×54

4. Divide, $113 \overline{)\,40228}$

5. 0.075^2

6. 997^2

7. 432×2002

8. Divide, $62 \overline{)\,19902}$

9. Work out 410.2367×201.2785 correct to 2 significant figures.

10. 0.48^3

11. What number, when multiplied by itself, is equal to 27×147 ?

12. Calculate, $729^2 - 271^2$

13. Express 45 as the difference of two square numbers that are integers in two different ways.

14. 0.0994×87.6

15. How many of the following positive integers are divisible by 24?

A $2^2 \times 3^2 \times 5^2 \times 7^3$ B $2^2 \times 3^2 \times 5^3 \times 7^2$ C $2^2 \times 3^3 \times 5^2 \times 7^2$ D $2^3 \times 3^2 \times 5^2 \times 7^2$

16. The sum of the areas of the squares on the sides of a right-angled isosceles triangle is 72 cm^2. What is the area of the triangle?

17. The integer n is the mean of the three numbers 17, 23 and $2n$. What is the digital root of n?

18. The positive integer n lies between 1 and 20. Rithik adds up all the integers from 1 to n inclusive. Tilly adds up all the numbers from $n+1$ to 20 inclusive. Their totals are the same. What is the value of n?

19. A frog sits on a lilly pad in a large pond covered in lillies. The frog jumps 1 metre, then 0.5 meters and then 0.25 metres; each time halving the distance of the previous jump. Theoretically, if the frog continues like this forever, how far will it go?

20. Find the equation of the straight line with gradient, 3, and that passes through the point (4, 2).

21. Find the equation of the straight line perpendicular to the line with equation, $3x+5y=23$ and which passes through the point (1, 5).

22. Find the equation of the straight line that passes through the points (2, 9) and (1, 2).

23. Two lines have equations, $2x+3y=15$ and $5x+4y=13$.
What is the position of their point of intersection?

24. Find the square root of the perfect square, 5776.

25. To 4 significant figures calculate the area of a rectangular sheet of metal measuring 1.324m by 10.06m.

26. How many non-recurring decimal digits are there in the decimal equivalent of $\frac{101}{475}$?

27. Calculate the decimal equivalent for $\frac{7}{29}$, correct to 6 decimal places.

28. Find the cube root of the exact cube, 238,328

29. Find the constants a and b given that,

$$(3x^2 - 2x + 7)(4x^2 + ax + b)$$
$$= 12x^4 + x^3 + 16x^2 + 25x - 14$$

30. Find the radius and position of centre of the circle with equation,

$$x^2 + y^2 - 10x + 6y - 15 = 0$$

31. Show that for all positive intergers n,

$$(3n+1)^2 - (3n-1)^2$$

is divisible by 12.

32. $f(x) = 2x^3 - 2x^2 - ax + a$

Given that $(x+2)$ is a factor of $f(x)$, find the value of the constant a.

33. Expand and simplify,

$$(2x^2 - 3x + 7)(3x^2 + 5x - 3)$$

34. Differentiate,

$$5xe^{2x}$$

35. Find $\frac{dy}{dx}$ given that,

$$y = \frac{5x+2}{3x+7}$$

36. Find the first three terms in the series expansion for,

$$\frac{8+x}{(2-x)^2}$$

37. Differentiate,

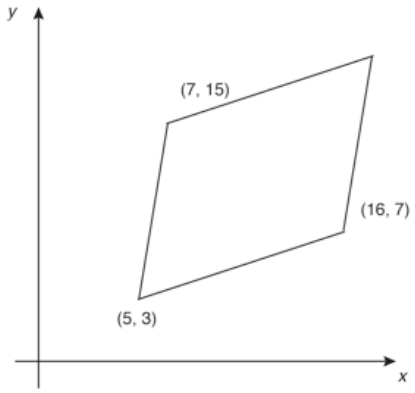
$$(3x^2+2x+1)^7$$

38. Evaluate,

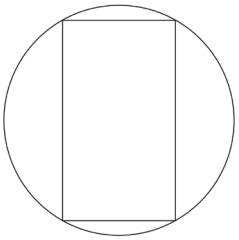
$$\tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{8}$$

39. Given the two triples, A) 4 3 5 and B) 12 5 13, find a triple for A + B.

40. Find the area of the parallelogram given the coordinates of three vertices as shown.



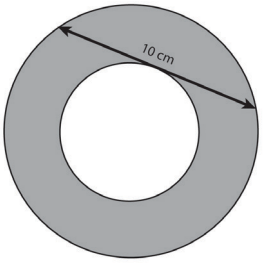
41.



The circle in the diagram has a radius of 6.5 cm. If the perimeter of the rectangle is 34 cm, what is its area?

42. Find the minimum value of k for which the line $y = 2x + k$ intersects with the curve, $y = x^2 - 4x - 5$.

43. Work out the area of the shaded region, leaving your answer in terms of π. The 10 cm line is tangent to the inner circle.



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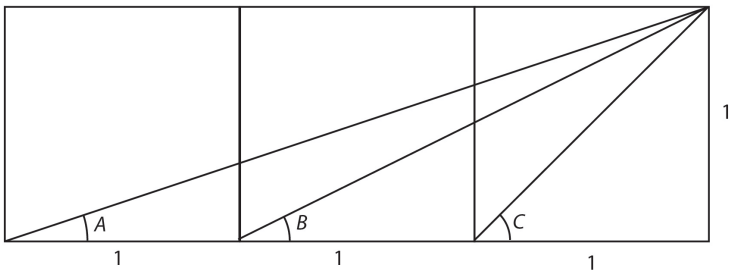


Find the sum of the three angles A, B and C.

45. Find the position of point A (5, 2) after it has been rotated about the origin through an angle of 60˚.

46. Find the shortest distance from the origin to the line with equation, $3x - 4y = 5$.

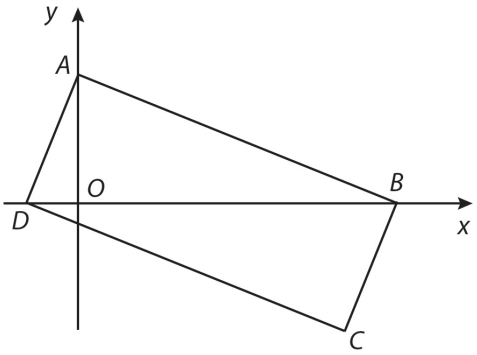
47. $99^3 + 3 \times 99^2 + 9 \times 33 + 1$

48. Find the area under the curve, with equation,

$$y = \frac{3x + 8}{(2x + 5)(x + 3)}$$

that lies above the x-axis and between the lines x = 0 and x = 2, leaving your answer in the form $a \ln b$.

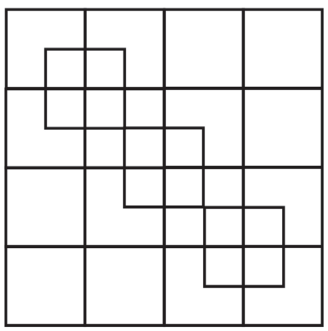
49.



The figure shows a rectangle, *ABCD*. The equation of the line *AB* is $2x + 5y = 10$. The point *A* lies on the y-axis and points *B* and *D* lie on the x-axis.

Work out the area of the rectangle.

50. How many squares are there?



IAVM

Sample IVMO Senior ANSWERS
Time allowed - 1 Hour

1.

$$\begin{array}{r} 984756 - 015244 \\ \times\ 999997 - 000003 \\ \hline 984753\ /\ 045732 \end{array}$$

All from 9 and the last from 10

2. Draw a circle round the number below that is divisible by 45.

637425

By elimination and retention

3.

$$\begin{array}{r} 52+02 \\ \times\ 54+04 \\ \hline 2)56\ /\ 08 \\ 28\ \ 08 \end{array}$$

All from 9 and the last from 10

4. Divide,

$$\begin{array}{r|l} 113 & 402/28 \\ \overline{1}\,3 & \overline{5}\,2 \\ & \quad 6\ \ 5 \\ & \quad\ \ \overline{7}\,8 \\ \hline & 4\,\overline{5}\,6/00 \end{array}$$

Transpose and apply

5. 0.075^2 0.005625

By one more than the one before

6. 997^2 994009

Whatever the extent of the deficiency, lessen it further and set up the square

7. 432×2002 864864

Transpose and apply

8. Divide,

$$\begin{array}{r|l} 6^2 & 19_1 9_1 0/2 \\ \hline & 3\ \ 2\ \ 1/0 \end{array}$$

Vertically and crosswise

9. Work out 410.2367 × 201.2785 correct to 2 significant figures.

$$\begin{array}{r} 410.2367 \\ \times\ 201.2785 \\ \hline 824_1 3 \end{array}$$

82000 (2 sf)

Vertically and crosswise

10. 0.48^3

$$\begin{array}{cccc} 64 & 128 & 256 & 512 \\ & 256 & 512 & \end{array}$$

$$\begin{array}{cccc} 110 & 5 & 9 & 2 \end{array}$$

0.110592

Proportionately

Ultimate and twice the penultimate

11. What number, when multiplied by itself, is equal to 27 × 147 ?

$3 \times 9 \times 3 \times 49 = 9^2 \times 7^2 = 63^2$

All the multipliers

12. Calculate,

$729^2 - 271^2$

$(729+271)(729-271) = 458{,}000$

By addition and subtraction

13. Express 45 as the difference of two square numbers that are integers in two different ways.

$45 = 5\times 9 = 7^2 - 2^2$

$= 3\times 15 = 9^2 - 6^2$

All the multipliers

By addition and subtraction

An additional answer can be obtained as,

$\left(\frac{45+1}{2}\right)^2 - \left(\frac{45-1}{2}\right)^2 = 23^2 - 22^2 = 45$

By addition and subtraction

14. 0.0994×87.6

$994 - 006$

$\times\ 876 - 124$

$870 / 744$

8.70744

All from 9 and the last from 10

15. How many of the following positive integers are divisible by 24?

A $2^2\times 3^2\times 5^2\times 7^3$ B $2^2\times 3^2\times 5^3\times 7^2$ C $2^2\times 3^3\times 5^2\times 7^2$ D $2^3\times 3^2\times 5^2\times 7^2$

$24 = 2\times 2\times 2\times 3$ ∴ One All the multipliers

16. The sum of the areas of the squares on the sides of a right-angled isosceles triangle is 72 cm². What is the area of the triangle?

$x^2 + x^2 + 2x^2 = 72$

$x^2 = 18$, Area $= 9$ cm^2

Transpose and apply

17. The integer n is the mean of the three numbers 17, 23 and $2n$. What is the digital root of n?

$\frac{17+23+2n}{3} = n, \quad n = 40, \quad DR = 4$

Specific and general

18. The positive integer n lies between 1 and 20. Rithik adds up all the integers from 1 to n inclusive. Tilly adds up all the numbers from $n+1$ to 20 inclusive. Their totals are the same. What is the value of n?

$S_{20} = \frac{20\times 21}{2} = 210$

$\frac{1}{2}S_{20} = 105 = \frac{n(n+1)}{2}, \quad n = 14$

When the total is the same, it is nought

19. A frog sits on a lilly pad in a large pond covered in lillies. The frog jumps 1 metre, then 0.5 meters and then 0.25 metres; each time halving the distance of the previous jump. Theoretically, if the frog continues like this forever, how far will it go?

$S_\infty = \frac{a}{1-r} = \frac{1}{1-0.5} = 2$

Specific and general

22. Find the equation of the straight line that passes through the points (2, 9) and (1, 2).

$$7x - y = 5$$

Transpose and apply

Product of the means

minus product of the extremes

23. Two lines have equations, $2x + 3y = 15$ and $5x + 4y = 13$.
What is the position of their point of intersection?

$$x = \frac{3\times13 - 4\times15}{3\times5 - 2\times4} = \frac{-21}{7} = -3$$

$$y = \frac{5\times15 - 2\times13}{7} = \frac{49}{7} = 7$$

Transpose and apply

Alternatively,

Adding $\Rightarrow\ 7x + 7y = 28\ \Rightarrow\ x + y = 4$

Subtracting $\Rightarrow\ -3x - y = 2$

Adding $\Rightarrow\ -2x = 6\ \Rightarrow\ x = -3\ \Rightarrow\ y = 7$

By addition and subtraction

24. Find the square root of the perfect square, 5776.

76

By the last digits

25. To 4 significant figures calculate the area of a rectangular sheet of metal measuring 1.324m by 10.06m.

$$\begin{array}{r} 1324 + 324 \\ \times\ 1006 + 006 \\ \hline 1331\ /_1 9_1 4_2 4 \\ \hline 13.32\ m^2 \end{array}$$

All from 9 and the last from 10

26. How many non-recurring decimal digits are there in the decimal equivalent of $\frac{101}{475}$?

$475 = 25\times19\ \therefore$ two

All the multipliers

27. Calculate the decimal equivalent for $\frac{7}{29}$, correct to 6 decimal places.

$0._1 24_1 1_2 3_2 793...$

0.241379

By one more than the one before

30. Find the radius and position of centre of the circle with equation,

$$x^2 + y^2 - 10x + 6y - 15 = 0$$

$(x-5)^2 - 25 + (y+3)^2 - 9 = 15$

$(5,-3) \quad r = 7$

By completion and non-completion

31. Show that for all positive intergers n,

$$(3n+1)^2 - (3n-1)^2$$

is divisible by 12.

$(3n+1+3n-1)(3n+1-3n+1)$

$= 12n$

By addition and subtraction

32. $f(x) = 2x^3 - 2x^2 - ax + a$

Given that $(x+2)$ is a factor of $f(x)$, find the value of the constant a.

$f(-2) = -16 - 8 + 2a + a = 0$

$a = 8$

When the total is the same, it is zero

Transpose and apply

33. Expand and simplify,

$$(2x^2 - 3x + 7)(3x^2 + 5x - 3)$$

$2x^2 - 3x + 7$

$\times \; 3x^2 + 5x - 3$

$6x^4 + x^3 + 0x^2 + 26x - 21$

Vertically and crosswise

34. Differentiate,

$$5xe^{2x}$$

$5x \quad e^{2x}$

$5 \quad 2e^{2x}$

$10xe^{2x} + 5e^{2x}$

Vertically and crosswise

Differential calculus

Particular and general

35. Find $\dfrac{dy}{dx}$ given that,

$$y = \frac{5x+2}{3x+7}$$

$\dfrac{29}{(3x+7)^2}$

Vertically and crosswise

Differential calculus

36. Find the first three terms in the series expansion for,

$$\frac{8+x}{(2-x)^2}$$

$2^{-2}(-x)^0 + -2.2^{-3}.(-x)^{-1} + -3.-2.2^{-4}\dfrac{(-x)^2}{2} + \cdots$

$\frac{1}{4} + \frac{1}{4}x + \frac{3}{16}x^2 + \cdots$

$8 + x$

$2 + \frac{9}{4}x + \frac{7}{4}x^2 + \cdots$

Differential calculus Vertically and crosswise

By one more than the one before

37. Differentiate,

$$(3x^2 + 2x + 1)^7$$

$7(3x^2 + 2x + 1)^6(6x + 2)$

Specific and general

38. Evaluate,

$$\tan^{-1}\tfrac{1}{2} + \tan^{-1}\tfrac{1}{5} + \tan^{-1}\tfrac{1}{8}$$

$$\begin{array}{r} 2 \quad 1 \\ +\ 5 \quad 1 \\ \hline 9 \quad 7 \\ +\ 8 \quad 1 \\ \hline 65 \quad 65 \end{array} \quad = 45^\circ$$

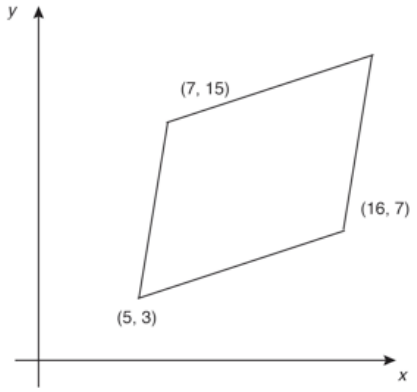
Vertically and crosswise

39. Given the two triples, A) 4 3 5 and B) 12 5 13, find a triple for A + B.

$$\begin{array}{r} 4 \quad 3 \quad 5 \\ +\ 12 \quad 5 \quad 13 \\ \hline 33 \ 56 \quad 65 \end{array}$$

Vertically and crosswise

40. Find the area of the parallelogram given the coordinates of three vertices as shown.



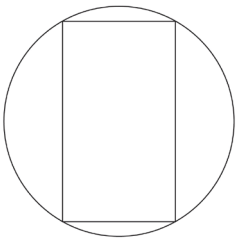
$$\begin{array}{r} (7,15) \quad (16,7) \\ -\ (5,3) \quad (5,\ 3) \\ \hline (2,12) \quad (11,4) \end{array}$$

$$= 132 - 8 = 124$$

Transpose and apply

Product of the means
minus product of the extremes

41.



The circle in the diagram has a radius of 6.5 cm. If the perimeter of the rectangle is 34 cm, what is its area?

$$6.5 \times 2 = 13, \quad 34 \div 2 = 17$$
$$13^2 = 5^2 + 12^2$$
$$\text{Area} = 5 \times 12 = 60 cm^2$$

When the totals are the same, it is zero

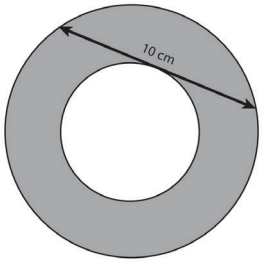
By inspection

42. Find the minimum value of k for which the line $y = 2x + k$ intersects with the curve, $y = x^2 - 4x - 5$.

$$x^2 - 4x - 5 = 2x + k$$
$$x^2 - 6x - (k + 5) = 0$$
$$b^2 - 4ac \geq 0$$
$$36 + 4k + 20 \geq 0$$
$$k \geq -14$$

When the totals are the same, it is zero

43. Work out the area of the shaded region, leaving your answer in terms of π. The 10 cm line is tangent to the inner circle.

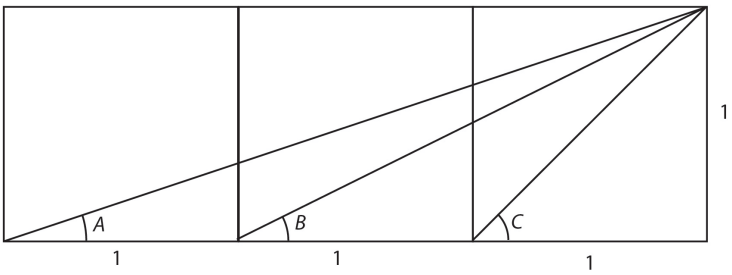


$R^2 - r^2 = 5^2$

Difference in areas $= \pi R^2 - \pi r^2 = \pi(R^2 - r^2) = 25\pi$

Transpose and apply

44.



Find the sum of the three angles A, B and C.

$$\begin{array}{r} 3\ \ 1 \\ +\ 2\ \ 1 \\ \hline 5\ \ 5 \\ +\ 1\ \ 1 \\ \hline 0\ \ 10 \end{array}$$

Vertically and crosswise

90°

45. Find the position of point A (5, 2) after it has been rotated about the origin through an angle of 60°.

$$\begin{array}{r} 5 \qquad 2 \\ +\ 1 \qquad \sqrt{3} \\ \hline 5-2\sqrt{3} \quad 5\sqrt{3}+2 \end{array}$$

Vertically and crosswise

46. Find the shortest distance from the origin to the line with equation, $3x - 4y = 5$.

$$\frac{|-5|}{\sqrt{3^2+4^2}} = 1$$

Transpose and apply

47. $99^3 + 3\times 99^2 + 9\times 33 + 1$

$99^3 + 3\times 99^2 + 3\times 99 + 1$ By inspection

$= (99+1)^3 = 1{,}000{,}000$

48. Find the area under the curve, with equation,

$$y = \frac{3x+8}{(2x+5)(x+3)}$$

that lies above the x-axis and between the lines x = 0 and x = 2, leaving your answer in the form $a \ln b$.

$$\frac{3x+8}{(2x+5)(x+3)} = \frac{1}{2x+5} + \frac{1}{x+3}$$

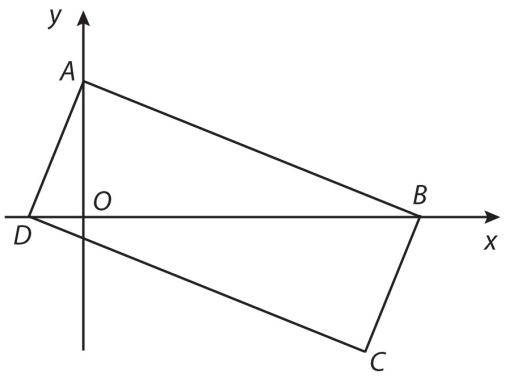
When the totals are the same, it is zero

Differential calculus

$$\text{Area} = \left[\tfrac{1}{2}\ln(2x+5) + \ln(x+3)\right]_0^2$$

$$= \tfrac{1}{2}\ln 9 + \ln 5 - \tfrac{1}{2}\ln 5 - \ln 3 = \tfrac{1}{2}\ln 5$$

49.



The figure shows a rectangle, *ABCD*. The equation of the line *AB* is $2x + 5y = 10$. The point *A* lies on the y-axis and points *B* and *D* lie on the x-axis.

Work out the area of the rectangle.

Transpose and adjust

A lies at (0, 2), *B* lies at (5, 0)

DA is $5x - 2y = -4$, *D* lies at $(\tfrac{-4}{5}, 0)$

By elimination and retention

$(\tfrac{-4}{5}, 0)$ $(5, 0)$

$-(0, 2)$ $(0, 2)$

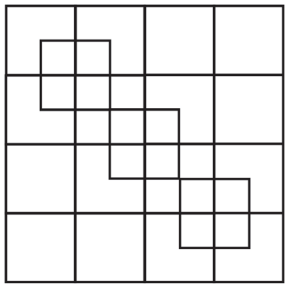
Product of the means

minus product of the extremes

$(\tfrac{-4}{5}, -2)$ $(5, -2)$

$$\text{Area} = \left|-10 - \tfrac{8}{5}\right| = 11\tfrac{3}{5}$$

50. How many squares are there?



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By elimination and retention